Experimental Determination of Projectile Range

T. J. Allen
Hobart and William Smith Colleges
Geneva, NY 14456 USA

September 15, 2001

Abstract

We report on the experimental determination of the range of a small metal projectile that is launched from a device that imparts a known initial speed and direction. We find that the equations for motion with constant acceleration can predict the range of the projectile to 3%, which is within the experimental uncertainty.

Introduction

Objects in free fall should move with constant downward acceleration equal to $g = 9.80$ m/s^2. The motion of a projectile can be predicted by using the kinematic equations for motion with constant acceleration.

In our experiment we used an ACME catapult that is calibrated to launch a 50 g projectile with an initial speed of 3.50 ± 0.05 m/s at an angle of 45°. We made 100 measurements of the range and we found that the range was $x = 1.212 \pm 0.002$ m. The theoretical range is 1.25 ± 0.04 m.

In the theory section we will show how one derives the range of a projectile from the initial velocity by using the equations for motion with constant acceleration. We present the analysis of our data in the third section. Finally, we present our conclusions and discussion in the last section.

Experimental Procedure

The equipment for this experiment consisted of the ACME catapult apparatus, which has an adjustable angle and a fixed initial projectile velocity, an small brass projectile, a table that can be adjusted to the same height as the released projectile, large sheets of paper and some carbon paper that we used to mark the impact point of the projectile, and a meter stick to measure the range of the projectile.

We adjusted the angle of the ACME catapult to 45°. We fired the projectile several times to find its approximate range and then we placed the paper under the carbon paper at the approximate impact point and fired the projectile one hundred times. Using the meter stick, we measured the range of each of the impacts.

Theory

A projectile moves with constant downward acceleration. The horizontal motion must therefore have constant horizontal speed. The equations that express this are

$$\begin{aligned} x &= x_0 + v_{0x}t \\ y &= y_0 + v_{0y}t - \frac{1}{2}gt^2 \ , \end{aligned} \tag{1}$$

where g is the acceleration of gravity, 9.80 m/s^2.

The components of the initial velocity are found by trigonometry from the initial speed and initial angle:

$$v_{0x} = v_0 \cos\theta \ ,$$

$$v_{0y} \;=\; v_0 \sin\theta \; . \tag{2}$$

In solving for the range of the projectile, it is easiest to find the time that the projectile is in the air. Because the projectile lands on the table at the same height as it was released, when the projectile returns to the table, its vertical velocity changes sign. Thus we have

$$v_y \;=\; v_{0y} - gt \;=\; -v_{0y} \; , \tag{3}$$

which we can solve for the time:

$$t \;=\; 2v_{0y}/g \;=\; 2v_0 \sin\theta/g \; . \tag{4}$$

Substitution of Eq. (4) into Eq. (1) yields the range:

$$x - x_0 \;=\; v_{0x}t \;=\; 2v_0^2 \cos\theta \, \sin\theta/g \;=\; v_0^2 \sin(2\theta)/g \; . \tag{5}$$

The last equality in Eq. (5) follows from the double angle formula for the sine function.

Analysis

We took one hundred measurements of the range using the ACME catapult. The average range was $\bar{x} = 1.212$ m and the standard deviation was $\sigma = 0.02$ m. The error in the mean is found by the relation [1]

$$\sigma_m = \frac{\sigma}{\sqrt{N}} \; . \tag{6}$$

In our case $N = 100$, which yields the result of our measurement of the mean range:

$$\bar{x} = 1.212 \pm 0.002 \; m \; . \tag{7}$$

Application of Eq. (5) with the stated initial velocity of the ACME apparatus,

$$v_0 \;=\; 3.50 \pm 0.05 \; m/s \; , \tag{8}$$

and the measured initial angle, $\theta = 45°$, yields a range of

$$x \;=\; \frac{(3.50 \; m/s)^2 \sin(2(45°))}{9.80 \; m/s^2} \;=\; 1.25 \; m \; . \tag{9}$$

To find the uncertainty in the theoretical range, we employ the formulas from the propagation of error appendix in Ref. [1].

$$\left(\frac{dx}{x}\right)^2 \;=\; \left(2\frac{dv_0}{v_0}\right)^2 + \left(\frac{dg}{g}\right)^2 \; . \tag{10}$$

Note that in Eq. (10) we assume that the angle θ is known precisely. If the angle were not known precisely, we would have to include its uncertainty as well. The formulas in Ref. [1] do not seem to apply directly to Eq. (5) because of the trigonometric function. If we assume that the error in g is in the least significant digit, we have $dg = 0.01$ m/s^2, and the error in the range follows from Eq. (10):

$$\left(\frac{dx}{1.25\ m}\right)^2 = \left(\frac{2(0.05\ m/s)}{3.50\ m/s}\right)^2 + \left(\frac{0.01\ m/s^2}{9.80\ m/s^2}\right)^2 , \tag{11}$$

or,

$$dx = 0.04\ m\ . \tag{12}$$

Conclusions

The first conclusion we must draw is that the theoretical and the experimental results agree. This is obvious because the experimental range, $x = 1.212 \pm 0.002$ m, overlaps the theoretical range, 1.25 ± 0.04 m, with the experimental average lying wholly within the theoretical range: $1.21 < 1.212 < 1.29$. More carefully, we note that the difference in the two values is smaller than the error in the difference:

$$(x_{\text{theory}} - x_{\text{expt}}) \ < \ \sqrt{(dx_{\text{theory}})^2 + (dx_{\text{expt}})^2}\ , \tag{13}$$

or

$$(1.25 - 1.212) \ < \ \sqrt{(0.04)^2 + (0.002)^2}\ . \tag{14}$$

The difference between the theoretical result and the experimental result is

$$\left(\frac{1.25 - 1.212}{1.25}\right) \ = \ 3.0\%\ . \tag{15}$$

Finally, we speculate upon sources of error in this experiment. The main source of error is likely to be the effect of air resistance, which is very difficult to account for theoretically at this level and is systematic in that it should decrease the range of all projectiles launched. Another source of error is likely to be the precision with which the projectile is aimed. We did not check that angle, except crudely with a protractor.

Despite these sources of error, the results do agree and this seems unlikely to be accidental. Our experiment confirms the applicability of the equations of projectile motion.

References

[1] L. Campbell, D. Spector, and T. Allen, *Physics 150 Laboratory Manual*, p. 57.