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Topological Mass Generation in 3+1 Dimensions

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Abstract

The four-dimensional theory of a 1-form abelian gauge field A coupled to a 2-form (antisymmetric tensor) potential B is studied. The two gauge invariances of the theory admit a coupling $mB \wedge F$ where F is the field strength ($F = \mathrm{d}A$) of A. It is shown that this theory is a unitary, renormalizable theory of a massive spin-one field with no additional degrees of freedom. In this sense it is a generalization to four dimensions of topological mechanisms in two dimensions (the Schwinger model) and three dimensions (Chern-Simons theory). The issue of spontaneous symmetry breaking is also examined.

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1. Introduction

Although gauge-invariance is usually associated with masslessness of the corresponding gauge field there are several mechanisms known for generating massive gauge fields consistent with gauge invariance. In two dimensions the Schwinger model of quantum electrodynamics coupled to a massless fermion yields a massive photon through the axial anomaly. In three dimensions the addition of a Chern-Simons term to the Lagrangian also results in a massive spin-1 vector field, with the mass arbitrary in the abelian case and quantized in the non-abelian case. The mass generation in these models may be viewed as resulting from non-trivial underlying topology [1].

In four dimensions a fundamentally different mechanism – the Higgs mechanism – is usually invoked to generate a mass gap whilst preserving the gauge invariance needed to ensure renormalizability of the theory. This involves arranging the arbitrary couplings of a scalar field coupled gauge-invariantly to the gauge field so that the vacuum expectation value of a physical scalar (the Higgs) is non-zero. The angular excitations of the scalar field combine with the two transverse degrees of freedom of the spin-one field to form a massive spin-one field. The arbitrariness of the scalar sector of the Higgs Lagrangian has long been a source of discomfort. In addition quadratic divergences in the mass renormalization of scalar fields force an unnatural fine-tuning of the parameters in the Lagrangian. There has been much effort to solve these latter two problems. One direction is to generate the scalar field dynamically as a bound state of a new set of strongly interacting fermionic matter fields. No completely realistic model of this sort has yet been found although it remains an attractive idea. Another direction is to supersymmetrize the model. This constrains the scalar couplings by embedding the scalar fields in a multiplet with fermion fields whose couplings are dictated by gauge invariance. Supersymmetry also eliminates the quadratic mass divergences. There is, however, no economy in the supersymmetric models as the Higgs field cannot be made the superpartner of any known fermion. The matter content of the theory is more than doubled and one must address the very difficult issue of realistic supersymmetry breaking to get a model consistent with the observed particle spectrum.

There is also known a topological-type gauge-invariant mechanism for generating mass for an abelian gauge field in four dimensions. It involves introducing a 2-form potential (Kalb-Ramond field) B [2] into the theory and coupling it to the gauge field through a $B \wedge F$ term. This theory has two types of gauge invariance and has therefore highly constrained couplings and is very geometrical. In this paper we examine this mechanism of mass generation and show that it is renormalizable and unitary and yields the spectrum of a single massive spin-one field with no remnant degrees of freedom.

The outline of the paper is as follows. In section 2 the theory of a Kalb Ramond field coupled to an abelian gauge field with the $B \wedge F$ mass term is introduced as a generalization of the Chern-Simons mass term in 2+1 dimensions. The detailed derivation of the propagator and the proof of renormalizability of the theory appear in section 3, and the Higgs mechanism is reviewed in section 4 for comparison. Finally in section 5 we look at the issue of symmetry breaking in the $B \wedge F$ theory.

2. A Topological Lagrangian in Four Dimensions

A well-known example of topological mass generation is the 2+1 dimensional Chern-Simons theory [3]. In this theory the relevant terms in the Lagrangian are

$$\mathcal{L}_{cs} = -\tfrac{1}{2} F \wedge {}^*F - \frac{\mu}{2} A \wedge F - A \wedge {}^*j, \tag{2.1}$$

where A is the gauge field, $F = \mathrm{d}A$ and $j^\mu = e\overline{\psi}\gamma^\mu\psi$. The equations of motion for A are

$$\mathrm{d}\,{}^*F = \mu F + {}^*j. \tag{2.2}$$

We also have the Bianchi identity

$$\mathrm{d}F = 0, \tag{2.3}$$

which can be thought of as the equation of motion for ${}^*F$. These two equations are called the London equations. Applying $\mathrm{d}\,{}^*$ to both sides of (2.2) we get

$$(\Box + \mu^2) F = -\mu\,{}^*j - \mathrm{d}j, \tag{2.4}$$

which is the equation of motion for a field F with mass μ in the presence of a coupled current j.

Now we look for a Lagrangian in four dimensions that gives us a generalization of the London equations. We will consider an abelian gauge field coupled to an antisymmetric tensor field. This coupling is a natural generalization of the 2+1 dimensional Chern-Simons term and, as we will see, also leads to a topological theory with an effective mass for the gauge field. Classically an antisymmetric tensor field is dual to a scalar – a propagating 2-form field has only one degree of freedom. This leads to the interesting possibility of a mechanism where the only degree of freedom of an uncharged 2-form field is 'eaten up' by the gauge field to give the latter an effective mass. First we make a digression to look at the free 2-form field and count its degrees of freedom.

Consider the Lagrangian for the free propagating antisymmetric tensor field,

$$\mathcal{L} = \tfrac{1}{2} H \wedge {}^*H, \tag{2.5}$$

where H is a 3-form derived from a 2-form potential B, $H = \mathrm{d}B$; in components $H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}$. This Lagrangian is invariant under the abelian gauge symmetry $B \to B + \mathrm{d}\Lambda$ where Λ is a 1-form.

The equations of motion derived from this Lagrangian are

$$\mathrm{d}\,{}^*H = 0. \tag{2.6}$$

We can solve this equation by the ansatz ${}^*H = \mathrm{d}\eta$ where η is a scalar field (0-form). Then the equation of motion for H is an identity for exterior derivatives and the Bianchi identity for H, $\mathrm{d}H = 0$, becomes the free Klein-Gordon equation for η. We can see via this 'dualization' that we are left with one degree of freedom for B. When we couple B to some other field, we cannot use this method of counting. If our Lagrangian, however, is invariant under the gauge transformation $B \to B + \mathrm{d}\Lambda$, we can see that out of the six components of B, only three are left free (we should note that there is a gauge transformation of Λ as well, $\Lambda \to \Lambda + \mathrm{d}\chi$). For the freely propagating B, the transverse modes are always zero, as can be seen from (2.6). For the Lagrangian we will consider now, this comment will still hold true.

Now we consider a Lagrangian where the B field couples to a massless gauge field and gives an effective mass to the latter. Such a Lagrangian is well-known [4,5],

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M + \mathcal{L}_I, \tag{2.7}$$

where

$$\begin{aligned} \mathcal{L}_G =& \tfrac{1}{2} H \wedge {}^*H - \tfrac{1}{2} F \wedge {}^*F + mB \wedge F, \\ \mathcal{L}_M =& \overline{\psi}(i\partial\!\!\!/ - m)\psi \\ \mathcal{L}_I =& - A \wedge {}^*j = -A_\mu j^\mu. \end{aligned} \tag{2.8}$$

Under the gauge variations $A \to A + \mathrm{d}\chi$ and $B \to B + \mathrm{d}\Lambda$ the first two terms of $\mathcal{L}$ remain invariant, while the last term has a variation $\mathrm{d}\Lambda \wedge F$, which is a total divergence. The action is, of course, invariant. (The condition of gauge invariance prevents terms like $B \wedge {}^*F$ from appearing in the Lagrangian.) The stress-energy tensor is the same as that in a theory of two free fields A and B because the interaction term can be written without invoking a metric. This is the reason the term is called 'topological'. The equations of motion are now modified,

$$\begin{aligned} \mathrm{d}\,{}^*H &= mF \\ \mathrm{d}\,{}^*F &= mH. \end{aligned} \tag{2.9}$$

Operating on both sides of the second equation with $\mathrm{d}\,{}^*$ and using the first equation, we get

$$(\square + m^2)F = 0. \tag{2.10}$$

This shows that the fluctuations of the field strength F are massive. It should be noted that H is also massive by the same analysis. On the other hand, one can choose a gauge in which the gauge field A itself has massive excitations and B (or H) is not in the spectrum any more. To show this, we solve the first equation of (2.9) by the ansatz ${}^*H = \mathrm{d}\eta + mA$. Then the second equation implies $\mathrm{d}\,{}^*\mathrm{d}A = (m\,{}^*\mathrm{d}\eta + m^2\,{}^*A)$. With a gauge choice ${}^*\mathrm{d}\,{}^*A = -m\eta$ (equivalently, $\mathrm{div}A = m\eta$) we get

$$(\square + m^2)A = 0, \tag{2.11}$$

which is the Klein-Gordon equation for a massive vector field A. We should note here that the massiveness is not a gauge artifact, because F obeys the massive Klein-Gordon equation [5] independent of gauge choice, as can be seen by substituting the second equation of (2.9) into the first. Another point of interest is that the field η is *not* a Higgs.

It should be noted here that a similar analysis in a different gauge shows that it is B which is massive in a different gauge. The mathematics is independent of whether we prefer to call A or B a massive field. In either case, however, we are left with a massive spin-one field (a massive antisymmetric tensor field has three degrees of freedom), and the physics may force us to choose between a massive gauge field and a massive axion.† In particular, it is more natural to view this as a theory with a massive gauge field in flat space in the presence of fermionic matter, while near a black hole with large closed strings present, this may be treated as a theory of a massive Kalb-Ramond field [6].

† The terminology is a little confusing here. A massive B has three degrees of freedom, while conventionally an axion is a scalar field that couples derivatively to other fields.

3. The Physical Propagator and Renormalizability

The full Lagrangian of a Kalb-Ramond field coupled to QED including fermion couplings and gauge fixing terms is

$$\mathcal{L} = \tfrac{1}{2}H \wedge {}^*H - \tfrac{1}{2}F \wedge {}^*F + mB \wedge F + \bar{\psi}(i\not{D} - M_0)\psi - \frac{1}{2\xi}(\mathrm{div}A)^2 - \frac{1}{2\zeta}(\mathrm{div}B)^2, \quad (3.1)$$

where D is the gauge covariant derivative. The bare propagators (fig. 1) are given by

$$\Delta^A_{\mu\nu}(k) = -\frac{g_{\mu\nu} - (1-\xi)k_\mu k_\nu/k^2}{k^2}, \quad (3.2)$$

$$\Delta^B_{\mu\nu,\rho\lambda}(k) = -\frac{g_{\mu[\rho}g_{\lambda]\nu} - (1-\zeta)g_{\mu[\rho}k_{\lambda]}k_\nu/k^2}{k^2}, \quad (3.3)$$

for A and B, respectively, and the $B-A$ vertex (fig. 2) is given by

$$\xi_{\mu\nu}V^{\mu\nu,\lambda}(k)\xi_\lambda = im\epsilon^{\mu\nu\rho\lambda}k_\rho\xi_{\mu\nu}\xi_\lambda, \quad (3.4)$$

where $\xi_{\mu\nu}$ and ξ_λ are polarization tensors for the B and A fields, respectively. (We will also write $S(p)$ for the fermion propagator $\frac{1}{\not{p}-M_0+i\epsilon}$.) The gauge we have chosen makes the subsequent calculations simple, but of course the same results will follow by choosing any other gauge. We find the 'combined' propagator (fig. 3) with the help of the above,

$$\widetilde{\Delta}^A_{\mu\nu}(k) = \Delta^A_{\mu\nu}(k) + \Delta^A_{\mu\mu'}(k)V^{\rho\lambda,\mu'}(k)\Delta^B_{\rho\lambda,\rho'\lambda'}(k)V^{\rho'\lambda',\nu'}(k)\Delta^A_{\nu'\nu}(k) + \cdots. \quad (3.5)$$

To compute this, we first note that

$$V^{\rho\lambda,\mu}(k)\Delta^B_{\rho\lambda,\rho'\lambda'}(k)V^{\rho'\lambda',\nu}(k) = -m^2(g^{\mu\nu} - k^\mu k^\nu/k^2) =: \theta^{\mu\nu}(k). \quad (3.6)$$

Then (3.5) implies that

$$\begin{aligned}\widetilde{\Delta}^A_{\mu\nu}(k) =& \Delta^A_{\mu\nu}(k) + \Delta^A_{\mu\mu'}(k)\theta^{\mu'\nu'}(k)\Delta^A_{\nu'\nu}(k) \\ &+ \Delta^A_{\mu\mu'}(k)\theta^{\mu'\nu'}(k)\Delta^A_{\nu'\rho}(k)\theta^{\rho\lambda}(k)\Delta^A_{\lambda\nu}(k) + \cdots \\ =& -\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2}\left(1 + \frac{m^2}{k^2} + \frac{m^4}{k^4} + \cdots\right) + \xi\frac{k_\mu k_\nu}{k^4} \\ =& -\frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - m^2} + \xi\frac{k_\mu k_\nu}{k^4}.\end{aligned} \quad (3.7)$$

After we set ξ to zero, we are left with a propagator that has a pole at $k^2 = m^2$. There also seems to be a pole at $k^2 = 0$, but this pole is a gauge artifact. This can be seen in the dual (scalar field) formulation very easily by choosing the gauge $\mathrm{div}A = m\eta$.

Now that we have obtained the 'combined' propagator, we look at the renormalizability of this theory. We note that this propagator has the same divergence structure as the photon propagator in usual quantum electrodynamics. This means that by arguments similar to those in the usual massless QED we can conclude that all divergences can be taken care of by introducing counterterms at the one-loop level. We proceed to calculate the one-loop divergent diagrams – these are essentially the one-loop divergent diagrams of usual QED with the photon propagator replaced by the 'massive' photon propagator $\widetilde{\Delta}^A_{\mu\nu}$.

The electron self-energy diagram (fig. 4) is

$$ie_0^2\Sigma(p) = \frac{e_0^2}{(2\pi)^4}\int d^4k\ \widetilde{\Delta}^A_{\mu\nu}\gamma^\mu S(p-k)\gamma^\nu. \tag{3.8}$$

This has the same degree of divergence as the corresponding diagram in usual QED. The divergent photon self-energy diagram (fig. 5) is proportional to $\widetilde{\Delta}^A_{\mu\alpha}(k)e_0^2\Pi^{\alpha\beta}(k)\widetilde{\Delta}^A_{\beta\nu}(k)$, with

$$\Pi^{\alpha\beta} \sim \frac{1}{(2\pi)^4}\mathrm{Tr}\int d^4p\ \gamma^\alpha S(p+k)\gamma^\beta S(p) \tag{3.9}$$

which is, in fact exactly the same expression as is obtained for the self-energy of the massless photon. The vertex correction comes from (fig. 6) and is given by

$$e_0^2\Lambda^\mu(p,p') \sim \frac{1}{(2\pi)^4}\int d^4k\ \gamma^\alpha S(p'-k)\gamma^\mu S(p-k)\gamma^\beta\widetilde{\Delta}^A_{\alpha\beta}(k). \tag{3.10}$$

This has no infra-red divergence, and the ultra-violet divergence is logarithmic as in usual quantum electrodynamics. The counterterms induced by these three diagrams cancel all higher order divergences, and the theory is renormalizable.

The gauge dependence of the counterterms can be seen by using arguments similar to the ones used for massive quantum electrodynamics [7]. Writing the gauge-fixed Lagrangian in terms of renormalized quantities,

$$\mathcal{L} = -\tfrac{1}{2}Z_3F\wedge {}^*F + \tfrac{1}{2}H\wedge {}^*H + mB\wedge F + \frac{1}{2\zeta}(\mathrm{div}B)^2 + \frac{1}{2\xi}(\mathrm{div}A)^2 - A\wedge {}^*j, \tag{3.11}$$

we get after some algebra

$$\begin{aligned} Z_3(\Box - \mathrm{d}\,{}^*\mathrm{d}\,{}^*)A + \frac{1}{\xi}\mathrm{d}\,{}^*\mathrm{d}\,{}^*A + m\,{}^*H - j &= 0,\\ \text{i.e.,}\qquad \frac{1}{\xi}\Box\ {}^*\mathrm{d}\,{}^*A &= 0, \end{aligned} \tag{3.12}$$

since $j = Z_2\overline{\psi}\gamma^\mu\psi$ is a conserved current. The Ward identity corresponding to (3.12) can be obtained by using

$$\frac{1}{\xi}\Box\partial\cdot A = \partial_\mu\frac{\Delta S}{\Delta A_\mu} - ie\overline{\psi}\frac{\Delta S}{\Delta\overline{\psi}} + ie\frac{\Delta S}{\Delta\psi}\psi \tag{3.13}$$

and

$$\langle 0|T\frac{\Delta S}{\Delta A_\mu}X|0\rangle = i\langle 0|T\frac{\Delta X}{\Delta A_\mu}|0\rangle,\ \text{etc.} \tag{3.14}$$

Then it follows that

$$\frac{1}{\xi}\Box\langle 0|T\partial\cdot A(x)X|0\rangle = i\frac{\delta_{gauge}}{\delta\omega(x)}\langle 0|TX|0\rangle, \tag{3.15}$$

where the gauge variation is computed using renormalized quantities.

This shows that $\partial\cdot A$ is a free field and therefore decouples from the theory. In particular, we can use transverse polarization ($\epsilon\cdot k = 0$) which picks out only the $k^2 = m^2$ pole in the 'photon' propagator (3.7) when we look at the gauge dependence of the S-matrix. The unitarity of the theory remains intact, as can be seen with the help of a simple argument. In the absence of the $B\wedge F$ term the ghosts decouple and the only propagating modes of the various fields are the transverse modes of A and the scalar mode of B. After the introduction of the interaction term we still have only these three modes propagating. As we said above, there is only one physical pole and therefore only one physical particle, the massive photon, with its three propagating modes. Therefore, no unphysical modes are propagated, and unitarity is unbroken.

4. Higgs Mechanism

The most popular mechanism of generating mass for a vector boson is the so-called Higgs mechanism. We will want to compare our proposed model with the Higgs model, so let us stop and examine the latter for a moment. We shall consider the Abelian Higgs model,

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\Phi)^\dagger(D_\mu\Phi) + V(\Phi) \tag{4.1}$$

where Φ is a complex scalar, D is the gauge covariant derivative $D_\mu = \partial_\mu - ieA_\mu$, $V(\Phi)$ is the usual Higgs potential $V(\Phi) = \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$ and we have suppressed the fermion couplings to Φ. This Lagrangian is invariant under the local gauge transformations

$$\begin{aligned}\Phi &\rightarrow \exp(-i\theta)\Phi\\ A_\mu &\rightarrow A_\mu - \frac{1}{e}\partial_\mu\theta\end{aligned} \tag{4.2}$$

It is useful to introduce 'angular' variables and parametrize Φ as

$$\begin{aligned}\Phi &= \frac{1}{\sqrt{2}}(\rho + v)\exp(-i\eta/v) \\ &= \frac{1}{\sqrt{2}}(v + \rho - i\eta + \cdots),\end{aligned} \tag{4.3}$$

where $v = (\mu^2/\lambda)^{\frac{1}{2}}$, $\mu^2 > 0$. Here we have made a choice of vacuum by choosing $\langle 0|\eta|0\rangle = 0$, and thus broken the global $U(1)$ symmetry.

The scalar meson ρ is usually called the 'Higgs' particle. Let us change the values of the parameters λ and μ such that $\mu, \lambda \to \infty$ but $v = (\frac{\mu^2}{\lambda})^{\frac{1}{2}}$ remains finite. We see that ρ becomes infinitely massive in this limit, and all terms involving ρ are therefore decoupled in an effective low-energy theory. The effective Lagrangian is then

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\partial_\mu\eta\partial^\mu\eta + \tfrac{1}{2}v^2e^2A_\mu A^\mu + evA_\mu\partial^\mu\eta \\ &= -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{M^2}{2}(A_\mu + \tfrac{1}{M}\partial_\mu\eta)^2.\end{aligned} \tag{4.4}$$

The 'gauge' nature of η is now evident. In particular, in the 'unitary' gauge $A^U_\mu = A_\mu + \frac{1}{M}\partial_\mu\eta$, the Lagrangian becomes

$$\mathcal{L} = -\tfrac{1}{4}F^U_{\mu\nu}F^{U\mu\nu} + \tfrac{M^2}{2}A^U_\mu A^{U\mu}. \tag{4.5}$$

The action resulting from the decoupling of the massive Higgs particle in (4.4) is the well-known Stückelberg action. Except for their zero modes, the classical fields η and $B_{\mu\nu}$ describe the same physics, as we will now argue.

In deriving the equation (2.11) † we made an ansatz

$$^*H = \mathrm{d}\eta + mA \tag{4.6}$$

to solve the first equation and then used the second equation of (2.9) to find the (second order) equation of motion for A

$$^*\mathrm{d}\,^*F = m(\mathrm{d}\eta + mA). \tag{4.7}$$

In this picture, we have essentially replaced the single degree of freedom of B by a scalar field η. The Bianchi identity, $\mathrm{d}H = 0$, must then be considered as a separate second order equation for η;

$$0 = \mathrm{d}H = \mathrm{d}\,^*(\mathrm{d}\eta + mA). \tag{4.8}$$

† Assuming trivial topology for spacetime

This displays the duality between the fields η and $B_{\mu\nu}$. The equation of motion for B becomes the defining relation for η. This defining equation is just a Bianchi identity for η. The Bianchi identity for B becomes the equation of motion for η. One would hope that the equations for η come from an action principle. In fact, because the field strength ${}^*H$ satisfies the massive Klein-Gordon equation, it should be that the action for the gauge invariant vector $\mathrm{d}\eta + mA$ is exactly the massive vector action

$$\mathcal{L}_{\eta,A} = -\tfrac{1}{2} F \wedge {}^*F + \tfrac{1}{2}(\mathrm{d}\eta + mA)^2. \tag{4.9}$$

That this is indeed the case is easy to verify, if we treat η as a Lagrange multiplier enforcing the constraint $\mathrm{d}H = 0$. Thus the field equations for the two Lagrangians are identical once the identification (4.6) is made.

A natural question to ask at this point is whether the theory described by (2.7) the same as a theory of spontaneously broken symmetry with an infinitely massive Higgs with the two-form B being no more than a convenient way of describing the angular part of Φ. The answer to this question is negative and the difference between the two theories can be seen at the quantum level. This is the question that we address now.

5. Symmetry Breaking

The Stückelberg Lagrangian (4.4) has a global symmetry, that of shifting $\eta \to \eta + c$ where c is a constant. This symmetry is present in any theory where a scalar couples to other matter only through derivative interactions, as does the axion. It is also present in the Higgs mechanism as a global phase rotation of the complex scalar Higgs field. This symmetry is 'spontaneously broken'. That is, while the Lagrangian has the symmetry, the Noether charge that generates it,

$$Q = \int \pi_\eta = \int \dot{\eta} = i(a^\dagger_{\vec{0}} - a_{\vec{0}}), \tag{5.1}$$

has an improper action on a ground state. The state $Q|0\rangle$, having one zero-momentum Goldstone boson, is not normalizable. Thus there is no meaning to the would-be unitary transformation $\exp(i\lambda Q)$ which would have related the two vacua $\langle 0|\eta|0\rangle = 0$ and $\langle\lambda|\eta|\lambda\rangle = \lambda$.

In dualizing a free 2-form theory to obtain a free scalar theory, we find two 'conserved currents', $j^\mu = {}^*H^\mu$ and $j^{\mu\nu\rho} = H^{\mu\nu\rho}$. The roles of the conservation laws of these currents as Bianchi identity or equations of motions are interchanged by the duality transformation.

The current j^μ is the Noether current for the symmetry $\eta \to \eta + c$ while the current $j^{\mu\nu\rho}$ leads to the Noether charge

$$Q^\Omega = \int_{{}^3\mathcal{M}} \Omega_{\mu\nu} j^{\mu\nu 0} = \int_{{}^3\mathcal{M}} \Omega_{\mu\nu} \Pi_B^{\mu\nu} \tag{5.2}$$

generating $B \to B + \Omega$. When Ω is exact, Q^Ω is a *constraint* and must annihilate all states. When Ω is closed, Q^Ω generates a symmetry of the action and the states must form a representation of this global $U(1)$ generator. The global gauge group is $[U(1)]^{\dim H^2({}^3\mathcal{M})}$. (Here we must absorb the extra Lorentz indices on the current $j^{\mu\nu\rho}$ so that the charge is well-defined. Then, strictly speaking, $\Omega_{\mu\nu} j^{\mu\nu\rho}$ is a conserved current only if $\partial_{[\rho}\Omega_{\mu\nu]} = 0$, that is, $[Q^\Omega, H] = \dot{Q}^\Omega \propto \int_{{}^3\mathcal{M}} \mathrm{d}\Omega \wedge {}^*H$.) If one analyzes dualization in the massive case, one finds that there are still Noether charges $Q = \int \pi_\eta = \int H$ and $Q^\Omega = \int \Omega_{\mu\nu}\Pi_B^{\mu\nu}$. The expression of the momenta in terms of fields and their time derivatives changes, while the canonical expressions do not. The axionic charge Q is conserved as long as there are appropriate boundary conditions on the momentum $\Pi_B^{\mu\nu}$. This follows from the equations of motion

$$\dot{Q} = [Q, H] \propto \int_{{}^3\mathcal{M}} \mathrm{d}\Pi_B = \int_{\partial\, {}^3\mathcal{M}} \Pi_B. \tag{5.3}$$

In the following we address the issue of the relation of these symmetries to the 2-form action and whether there is global symmetry breaking in the theory written with a 2-form.

Following [8] we may examine the dualization in path integral language. These authors start with a field strength formulation and insert the condition the $H = \mathrm{d}B$ locally by using a δ function,

$$\mathcal{Z} = \int \mathcal{D}H\, \delta[\mathrm{d}H] \exp{(i \int \tfrac{1}{2} H \wedge {}^*H)}. \tag{5.4}$$

Then a scalar is introduced to exponentiate the argument of the δ function,

$$\mathcal{Z} = \int \mathcal{D}\eta \mathcal{D}H \exp{(i \int \tfrac{1}{2} H \wedge {}^*H + \eta \wedge \mathrm{d}H)}. \tag{5.5}$$

From here it is obvious that η is canonically conjugate to the spatial part of H, H_{ijk}. A canonical version of this analysis appears in [9]. Writing $H = q(t)\omega_{vol} + \widetilde{H}$, we find explicitly that the constant mode of η is conjugate to $q(t)$. The charge $Q^\Omega = \int \Omega_{\mu\nu}\Pi_B^{\mu\nu}$ generates shifts in $q(t)$ if $\mathrm{d}\Omega = \omega_{vol}$, but then Q^Ω is not a conserved charge and does not generate a symmetry of the action.

When $\mathrm{d}\Omega = 0$ one finds that $[Q, Q^\Omega] = i\int \mathrm{d}\Omega = 0$. These two charges have actions which commute. When written in the dual variables of the two-form theory, the axionic charge $Q = \int_{^3\mathcal{M}} H$ may be seen to have *no* effect upon the dynamical variables,

$$\begin{aligned}\delta_{ax}B_{\mu\nu}(x) =&[B_{\mu\nu}(x), Q] \equiv 0,\\ \delta_{ax}\Pi^{\mu\nu}_B(x) =&[\Pi^{\mu\nu}_B(x), Q] \propto \int \epsilon^{0\mu\nu\rho}\partial_\rho\delta(x-y)d^3y = 0.\end{aligned} \tag{5.6}$$

Thus it does not generate any nontrivial symmetry of the action and there is no symmetry breaking in the $B_{\mu\nu}$ system corresponding to the symmetry breaking in the η system.

The preceding discussion is for space-times which have spatial sections that are topologically trivial, $H^2(\,^3\mathcal{M}) = 0$. Q^Ω above was just the generator of gauge transformations $B \to B + \Omega$ where $\Omega = \mathrm{d}\lambda$ $\;(H^2(\,^3\mathcal{M}) = 0$ iff $\mathrm{d}\Omega = 0 \Rightarrow \Omega = \mathrm{d}\lambda)$. When the space-time is nontrivial more interesting things can happen. In nontrivial space-times, (such as axionic black hole spacetimes [6] or when $\,^3\mathcal{M} = \mathbf{R}^3\backslash\{0\}$) the axionic charge is, strictly speaking, *not* $\int_{^3\mathcal{M}} H = \oint_{\partial\,^3\mathcal{M}} B$, since the boundary includes a 2-sphere around $r = 0$. The suitable axionic charge is $Q_\infty = \oint_{\partial\,^3\mathcal{M}_\infty} B$, the integral only around the boundary component at spatial infinity. Now $[Q_\infty, Q^\Omega] = i\oint_{\partial\,^3\mathcal{M}_\infty}\Omega$. The axionic charge Q_∞ is changed by Q^Ω whenever $\Omega \in H^2(\,^3\mathcal{M})$ is non-trivial. This behavior is reminiscent of large gauge transformations in Yang-Mills theories where gauge transformations which cannot be built up from infinitesimal gauge transformations change the 'winding number' of the vacuum. Here the analogous 'winding number' is the axionic charge $Q_\infty = \oint_{\partial\,^3\mathcal{M}_\infty} B$. We conclude then that in the $B_{\mu\nu}$ variables there is no remnant of the scalar shift $\eta \to \eta + c$. This is particularly evident in topologically trivial spacetimes, where one is always able to shift $\eta \to \eta + c$ but there are no 'large' gauge transformation of $B_{\mu\nu}$ to be made. In spacetimes with non-trivial $H^2(\,^3\mathcal{M})$ we expect that the vacuum structure will be similar to that of Yang-Mills theories.

6. Conclusion

To summarize, we have shown that the topological mass term for an abelian two-form coupled to massless QED with unitary and renormalizable matter couplings becomes massive QED and remains unitary and renormalizable.

Furthermore, in spacetimes with topologically trivial spatial sections there is no spontaneous breaking of any global symmetry, as there is in the abelian Higgs model. We have shown that the two currents which are dual to each other are the generators of global gauge transformations and the generator of the global phase rotation in the Higgs (to be precise, infinitely massive Higgs, or the Stückelberg) model and that these symmetries are not equivalent.

It seems possible that this mechanism can be generalized to non-abelian gauge fields in 3+1 dimensions using the non-abelian generalization of the Kalb-Ramond gauge invariance due to Rajeev [4]. Such a mechanism would obviate the need for a Higgs boson, left over from the process of giving mass to the gauge fields (via the usual Higgs mechanism) but would not address the independent problems of giving masses to the quarks and leptons and of chiral symmetry breaking. In passing, we note also that the theory consisting of the $B \wedge F$ interaction alone is an interesting example of an exactly soluble four-dimensional topological field theory [10].

A first announcement of some of the results presented in this paper appeared in [11].

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