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Simpson's Neutrino and the Singular See-Saw

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Abstract

We derive explicit forms for the neutrino and lepton "mixing-matrices" which describe the generic singular see-saw model. The dependence on the hierarchy parameter is contrasted with the non-singular case. Application is made to Simpson's 17 *keV* neutrino.

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1. Introduction

Very recently, the deduction from the measured end point spectrum of nuclear beta decay by Simpson[1] of about a 10% (in amplitude) "contamination" of the electron neutrino with a different neutrino of mass 17 keV has been supported by other groups.[2] These are difficult experiments but, if correct, they have very profound implications for our understanding of electro-weak interactions. Indeed, there have already been many interesting theoretical reports[3−9] on the matter.

The new observations seem puzzling according to usual theoretical models and expectations. In the first place the neutrino masses are expected to be in the range 10^{-4} to 10^{-2} eV in order to explain the observed solar neutrino flux with the resonant oscillation mechanism.[10] The 17 keV mass is far enough away from these values to suggest the existence of another new scale in weak interaction physics. Secondly, when one recognizes that the general (or "natural") massive neutrino in gauge theories is of the two-component "Majorana" type, it becomes a puzzle to understand why a 17 keV neutrino mixing with ν_e at the 10% level does not strongly violate the experimental bound[11] on neutrinoless double beta decay ($\beta\beta_{0\nu}$). The only way out would be to have two Majorana neutrinos whose contributions cancel each other. If the two are degenerate, or nearly so, they effectively combine to be one of Dirac type. In the initial attempts[12] to explain Simpson's neutrino it was postulated that the three two-component fields for the three generations should be regarded as one very light ν_e and nearly degenerate ν_μ and ν_τ at 17 keV. This is notsuitable† for the current explanation[10] of the solar neutrino flux which requires two light neutrinos. The minimal set of two-component neutrino fields required is thus four — two for solar flux and two for $\beta\beta_{0\nu}$. But, considering just four neutrino fields is unaesthetic in the sense that it disagrees with the parallel generation structure observed for the other fermi fields in nature. Ordinarily, one's intuition about massive neutrinos is guided by the see-saw mechanism[15] in which two two-component fields for each generation are assumed and where the 6×6 mass matrix has the structure

$$M = M^T = \begin{pmatrix} 0 & \epsilon D \\ \epsilon D^T & M_H \end{pmatrix}. \tag{1.1}$$

The "heavy" mass matrix M_H has a scale corresponding to some "new physics." There are three superheavy neutrinos and three whose masses are suppressed from a "typical" fermionic scale ϵD by the dimensionless hierarchy parameter ϵ, the effective light 3×3 neutrino mass matrix being $-\epsilon^2 D M_H^{-1} D^T$. (Note that D and M_H are being taken of the same order so that ϵ is explicitly displayed.)

† However such an explanation might be viable if one were able to explain[13] the reduction of the solar neutrino flux by neutrino flavor-spin rotation[14] in the sun's magnetic field.

One option for explaining Simpson's neutrino in a parallel generation framework is simply to give up the "new physics" scale by setting $M_H = 0$. Then all three neutrinos would be of Dirac type. This does not explain why ν_τ is so much heavier than ν_e and ν_μ. Several years ago, in a study of the way M_H could be related to the mass matrices of the charged fermions, it was found[16] that M_H tended to come out close to a singular matrix (of rank 2). The exactly singular case was noted to lead to a rather unusual pattern of neutrino masses which was advocated as an interesting possibility if ν_τ was found to be substantially heavier than ν_e and ν_μ. In this pattern there are only two superheavy neutrinos with masses of order M_H, two very light Majorana neutrinos (ν_e and ν_μ presumably) with masses of order $\epsilon^2 M_H$ and, most interestingly, an effective Dirac neutrino of mass order ϵM_H (identified as 17 keV). This is just the pattern needed and has very recently been discussed by other investigators.[3,4]

In this note we will further discuss some technical aspects of the singular see-saw mechanism. In particular we would like to obtain explicitly the unitary 6×6 transformation matrix U connecting the "bare" and physical (mass diagonal) neutrino fields for a generic singular see-saw model:

$$\rho = U\nu. \tag{1.2}$$

Here ρ and ν are respectively the bare and physical column vectors of two-component fields. Furthermore

$$U^T M U = \text{ diagonal.} \tag{1.3}$$

The transformation matrix U is central to the discussion of the properties of the various neutrinos. Of special interest are the possible decay modes of the 17 keV neutrino. Astrophysical and cosmological criteria necessitate a relatively quick decay. One promising mode is a decay such as $\nu_\tau \to \nu_\mu + J$, where J is the Majoron,[17] a true Goldstone boson associated with spontaneous breakdown of lepton number. A naïve estimate would suggest that the amplitude for this process goes as ϵ^2, which is sufficiently rapid for $\epsilon \sim m(\nu_\mu)/m(\nu_\tau) \sim 10^{-7}$. However there is a subtlety due to the Goldstone nature of J and it turns out[18] that for the non-singular see-saw the amplitude is suppressed to order ϵ^4, which is not sufficient. We will explicitly show how the singular see-saw restores the ϵ^2 order for the amplitude. Similarly the amplitude for $\nu_\tau \to 3\nu$ due to Z boson exchange, which is of order ϵ^2 for the non-singular see-saw,[18] will be shown to be enhanced to order ϵ in the singular case.

The transformation matrix is derived in Sec. 2 and applied to the Majoron decay modes in Sec. 3. Section 4 is concerned with other decay modes while Sec. 5 presents the parametrization of the lepton mixing matrix for the singular see-saw.

2. Transformation Matrix

In the general singular see-saw model ϵD in eq. (1.1) is arbitrary and $M_H = R^{-1}\widetilde{\mathcal{M}}R$ where $\widetilde{\mathcal{M}} = \mathrm{diag}(0, M_1, M_2)$. (One might think of setting $R = 1$ but if a particular model is specified it might be more convenient to allow for $R \neq 1$.) We shall specialize to real M for simplicity, the generalization to complex M being straightforward. Thus R is an ordinary 3×3 rotation matrix. Our approach will be to make a succession of similarity transformations. We start by writing

$$\begin{pmatrix} 0 & \epsilon D \\ \epsilon D^T & R^T\widetilde{\mathcal{M}}R \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & R^T \end{pmatrix} \begin{pmatrix} 0 & \epsilon DR^T \\ \epsilon RD^T & \widetilde{\mathcal{M}} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & R \end{pmatrix}. \tag{2.1}$$

Notice that $DR^T = (\,\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}\,)$ is an arbitrary real 3×3 matrix. The crucial step is the exact diagonalization of the upper left 4×4 sub-block in the central matrix on the right side. This yields

$$\begin{pmatrix} 0 & \epsilon DR^T \\ \epsilon RD^T & \widetilde{\mathcal{M}} \end{pmatrix} = \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \epsilon A & \epsilon B \\ \epsilon B^T & \mathcal{M} \end{pmatrix} \begin{pmatrix} V^T & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.2}$$

where the matrix on the left is partitioned into 3×3 sub-blocks while the matrices on the right are partitioned in a $(4, 2)$ pattern. The diagonal matrix of eigenvalues $\epsilon A = \mathrm{diag}(0, 0, \epsilon|\mathbf{a}|, -\epsilon|\mathbf{a}|)$ shows that there are two entries with the same magnitude $\epsilon|\mathbf{a}|$; these are essentially the two "Majorana" components of the effective Dirac neutrino. Note now that $\mathcal{M}$ without the tilde is a diagonal 2×2 matrix . The 4×4 diagonalizing matrix V is

$$V = \begin{pmatrix} \widehat{\mathbf{n} \times \mathbf{a}} & (\widehat{\mathbf{n} \times \mathbf{a}}) \times \mathbf{a} & \hat{\mathbf{a}}/\sqrt{2} & \hat{\mathbf{a}}/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \tag{2.3}$$

in which $\mathbf{n}$ is an arbitrary unit vector. Three independent real numbers are needed to specify V: two for $\hat{\mathbf{a}}$ and one for $\mathbf{n}$ (since only the components of $\mathbf{n}$ perpendicular to $\hat{\mathbf{a}}$ are relevant). In addition

$$\mathcal{M} = \mathrm{diag}(M_1, M_2), \qquad B = V \begin{pmatrix} \mathbf{b} & \mathbf{c} \\ 0 & 0 \end{pmatrix}. \tag{2.4}$$

Next, we approximately block-diagonalize the central matrix on the right side of eq. (2.2):

$$\begin{pmatrix}\epsilon A & \epsilon B\\ \epsilon B^T & \mathcal{M}\end{pmatrix}\approx \begin{pmatrix}1 & \epsilon B\mathcal{M}^{-1}\\ -\epsilon\mathcal{M}^{-1}B^T & 1\end{pmatrix}\begin{pmatrix}\epsilon A-\epsilon^2 B\mathcal{M}^{-1}B^T & 0\\ 0 & \mathcal{M}\end{pmatrix}\begin{pmatrix}1 & -\epsilon B\mathcal{M}^{-1}\\ \epsilon\mathcal{M}^{-1}B^T & 1\end{pmatrix}. \tag{2.5}$$

The 4×4 matrix $-\epsilon^2 B\mathcal{M}^{-1}B^T$ supplies masses to ν_e and ν_μ of order $\epsilon^2\mathcal{M}$ and slightly splits the two Majorana components of the 17 keV Dirac neutrino by $\epsilon^2[(B\mathcal{M}^{-1}B^T)_{33}+(B\mathcal{M}^{-1}B^T)_{44}]$. We define W as the 4×4 matrix which accomplishes the final diagonalization

$$W^T(\epsilon A-\epsilon^2 B\mathcal{M}^{-1}B^T)W = m = \mathrm{diag}(m_1, m_2, m_3, m_4). \tag{2.6}$$

Multiplying together the four transformation matrices in eqs. (2.1), (2.2), (2.5) and (2.6) yields for the transformation matrix defined by eq. (1.3):

$$U \approx \begin{array}{cc} & \begin{array}{cc} 3 & 3 \end{array} \\ \begin{array}{c} 3 \\ 3 \end{array} & \begin{pmatrix}1 & 0\\ 0 & R^T\end{pmatrix}\end{array} \begin{array}{cc} & \begin{array}{cc} 4 & 2 \end{array} \\ \begin{array}{c} 4 \\ 2 \end{array} & \begin{pmatrix} VW & \epsilon VB\mathcal{M}^{-1}\\ -\epsilon\mathcal{M}^{-1}B^T W & 1\end{pmatrix}\end{array}, \tag{2.7}$$

where we must **caution** the reader that the first matrix is partitioned as $(3,3)$ and the second as $(4,2)$. Eq. (2.7) is basic for the following. At present only W is not explicit.

3. Majoron decay modes

It was pointed out[16,3] that introducing a Majoron field J could promote the viability of the present scheme by providing a rapid decay mode[17] such as $\nu_\tau \to \nu_\mu + J$. We will focus our attention on the simplest Majoron model — the singlet Majoron.[17] If the ordinary non-singular see-saw is employed, the decay is suppressed by the Goldstone nature of J, unfortunately. The crucial point is to see how the singular see-saw enhances this mode. For this purpose we will simplify the calculation by simplifying R to have the form

$$R \to \begin{pmatrix} 1 & 0 \\ 0 & \tilde{R} \end{pmatrix}, \tag{3.1}$$

so that eq. (2.7) is neatly partitioned in a $(4,2)$ pattern.

$$U = \begin{array}{c c} & \begin{array}{cc} 4 & 2 \end{array} \\ \begin{array}{c} 4 \\ 2 \end{array} & \begin{pmatrix} U_a & U_b \\ U_c & U_d \end{pmatrix} \end{array} \approx \begin{pmatrix} VW & \epsilon VB\mathcal{M}^{-1} \\ -\epsilon\tilde{R}^T\mathcal{M}^{-1}B^TW & \tilde{R}^T \end{pmatrix}. \tag{3.2}$$

The Majoron Yukawa interaction is given by eq. (3.6) of Ref. 18 (noting the $(4,2)$ partition now):

$$\mathcal{L}_Y = \frac{iJ}{2x}\nu^T\sigma_2 \begin{pmatrix} U_c^T\tilde{R}^T\mathcal{M}\tilde{R}U_c & U_c^T\tilde{R}^T\mathcal{M}\tilde{R}U_d \\ U_d^T\tilde{R}^T\mathcal{M}\tilde{R}U_c & U_d^T\tilde{R}^T\mathcal{M}\tilde{R}U_d \end{pmatrix} \nu + \text{h.c.}, \tag{3.3}$$

where $x \sim \mathcal{O}(M) \sim 170\,GeV$ is the vacuum value of the singlet Higgs field in this model. The 4×4 submatrix for the Majoron Yukawa coupling constants of the four "light" neutrinos is given by the upper left sub-block in (3.3); using (3.2) and (2.6) we obtain

$$\frac{1}{x}U_c^T\tilde{R}^T\mathcal{M}\tilde{R}U_c \approx \frac{-m}{x} + \frac{\epsilon}{x}W^TAW. \tag{3.4}$$

At this stage we may recover the old result[18] for the non-singular see-saw by noting that the calculation in that case is structurally the same as the present one except that the second term on the right side of eq. (3.4) is absent. Since the first term is diagonal, there is then no off-diagonal coupling to mediate $\nu_\tau \to \nu_\mu + J$, for example! To treat the second term in (3.4) we approximately diagonalize eq. (2.6) at the 2×2 block level to find

$$W \approx \begin{pmatrix} 1 & \epsilon Q_b\tilde{A}^{-1} \\ -\epsilon\tilde{A}^{-1}Q_b^T & 1 \end{pmatrix} \begin{pmatrix} Z & 0 \\ 0 & 1 \end{pmatrix}, \tag{3.5}$$

wherein

$$\begin{aligned} \tilde{A} &= \text{diag}(|\mathbf{a}|, -|\mathbf{a}|), \\ -B\mathcal{M}^{-1}B^T &\equiv \begin{pmatrix} Q_a & Q_b \\ Q_b^T & Q_d \end{pmatrix} \end{aligned} \tag{3.6}$$

and Z^TQ_aZ is a diagonal 2×2 matrix. The second term in eq. (3.4) then becomes approximately

$$\frac{\epsilon}{x} W^T A W \approx \frac{\epsilon}{x} \begin{pmatrix} 0 & -\epsilon Z^T Q_b \\ -\epsilon Q_b^T Z & \tilde{A} \end{pmatrix}. \tag{3.7}$$

Finally, the Yukawa couplings for $\nu_{3,4} \to \nu_{1,2} + J$ are explicitly given by the off-diagonal sub-block in (3.7) and are immediately read off to be of order ϵ^2. This is in contrast to the ϵ^4 amplitude[18] in the non-singular case. Numerically,

$$\Gamma(\nu_\tau \to \nu_\mu J) \approx \frac{(\epsilon^2)^2}{16\pi} m(\nu_\tau) \approx 0.51 \times 10^{-10} \mathrm{sec}^{-1}. \tag{3.8}$$

This is in agreement with the cosmological density bound[19] for a 17 keV neutrino

$$\Gamma_{\mathrm{total}}(\nu_\tau) > 0.3 \times 10^{-11} \mathrm{sec}^{-1}. \tag{3.9}$$

It may be of interest to note that the amplitude for one of the superheavy neutrinos to decay into a Majoron and a light neutrino is determined by the upper right rectangular sub-block in (3.3) and is of order ϵ. The partial width for this mode is then about $10^{11} sec^{-1}$.

4. Other decay modes

In the non-singular see-saw model the amplitude for a decay like $\nu_\tau \to \nu_e + 2\nu_\mu$ mediated by Z boson exchange is[18] of order ϵ^2. We will see that this amplitude is also enhanced for the singular case. The interaction term for the neutrinos and the Z in the SU(2)×U(1) theory Lagrangian is $(ig'/2\sin\theta_W) Z_\mu \bar{\nu}_L \gamma^\mu P \nu_L$ where $\nu_L = \begin{pmatrix} \nu \\ 0 \end{pmatrix}$ in a γ_5-diagonal representation of the Dirac matrices and P is the 6×6 matrix

$$P_{\alpha\beta} = \sum_{a=1}^{3} U^\dagger_{\alpha a} U_{a\beta}. \tag{4.1}$$

Here, of course, we are interested in the restriction of $P_{\alpha\beta}$ to the first four "light" neutrinos. We note from (3.2) that the upper left sub-block VW of U is approximately unitary by itself so that

$$P_{\alpha\beta} \approx \delta_{\alpha\beta} - (VW)_{4\alpha}(VW)_{4\beta} \quad \text{for } \alpha, \beta = 1, \ldots, 4. \tag{4.2}$$

In the non-singular case the second term of (4.2) is absent so $P_{\alpha\beta}$ is purely diagonal to lowest order. To see that the amplitude for, say, the $\nu_\tau \to \nu_e$ transition is enhanced to order ϵ for the singular see-saw note that $P_{13} \approx -(VW)_{43}(VW)_{41}$ with $(VW)_{41} \approx (W_{31} - W_{41})/\sqrt{2} \sim \epsilon$ [using eqs. (2.3) and (3.5)]. Similarly $(VW)_{43} \approx 1/\sqrt{2}$. Numerically, for a typical decay, we find

$$\Gamma(\nu_\tau \to \nu_e \nu_\mu \nu_\mu) \approx \epsilon^2 \left[\frac{m(\nu_\tau)}{m(\mu)}\right]^5 \Gamma(\mu \to e \bar{\nu}_e \nu_\mu) \approx 10^{-27} \mathrm{sec}^{-1}. \tag{4.3}$$

So even though this decay rate is enhanced in the singular see-saw it is still much too small by itself to satisfy the cosmological bound (3.9). It is possible to enhance the 3ν decay mode further[5,6] if one includes the effect of exchanging new neutral Higgs bosons. For comparison with (3.8) and (4.3) we must also mention the more "standard" decay modes such as $\nu_\tau \to \nu_\mu + \gamma$. These are limited by various astrophysical arguments[20] on the number of photons allowed in various circumstances to

$$\frac{\Gamma(\nu_\tau \to \nu\gamma)}{\Gamma(\nu_\tau \to \text{no photons})} \lesssim 10^{-5}. \tag{4.4}$$

We estimate[21] the partial width for photon decay as

$$\begin{aligned} \Gamma(\nu_\tau \to \nu_e \gamma) &\approx \frac{\alpha G_F^2}{128\pi^4} \cdot \frac{9}{16} \cdot m^5(\nu_\tau) |K_{\tau\nu_e}|^2 \left[\frac{m(\tau)}{m(W)}\right]^4 \\ &\approx 10^{-25} \mathrm{sec}^{-1}, \end{aligned} \tag{4.5}$$

where $\alpha \approx 1/137$, G_F is the Fermi constant and $K_{\tau\nu_e} \approx 0.1$ is an element of the leptonic Kobayashi-Maskawa matrix. Comparing the ratio of eq. (3.8) to eq. (4.5) with (4.4) it is clear that, in the singular see-saw Majoron model, the invisible Majoron decay mode is highly dominant over the radiative decays, so there are no astrophysical difficulties.

5. Lepton mixing matrix

For discussing the ordinary, W boson mediated, weak interactions it is of course crucial to know the leptonic analog of the K-M matrix which we denote as K. It enters into the interaction Lagrangian as:

$$\mathcal{L}_W = \frac{ig}{\sqrt{2}} W_\mu^- \bar{e}_L \gamma^\mu K \nu_L + \text{h.c.} \ . \tag{5.1}$$

Here the three physical charged leptons e_L are related to the "bare" fields E_L as $E_L = \Omega e_L$. K is a rectangular matrix:

$$K_{b\alpha} = \sum_{c=1}^{3} \Omega^\dagger_{bc} U_{c\alpha}, \tag{5.2}$$

having, to start with, three rows and six columns. However, for discussing processes mediated by (5.1) and involving the four light two-component neutrino fields we may neglect the two superheavy neutrinos; then K has three rows and four columns. The parametrization of such a K which involves three SU(2) doublet neutrinos and one SU(2) singlet neutrino may be slightly unfamiliar. In ref. 22 such parametrizations were discussed for the general case of n doublet and m singlet neutrinos [(n, m) model]. For our present $(3, 1)$ case there are actually six angles and six CP violation phases required in general. Since we are assuming K to be real there are just six angles which, as we will now show, get reduced to three at the needed zeroth order in ϵ description of the singular see-saw. Think of substituting the U in eq. (3.2), truncated to the 4×4 light subspace, into eq. (5.2). The main structure is provided by V given in eq. (2.3). Multiplying by W (see eq. (3.5)) on the right will, to zeroth order in ϵ, leave the third and fourth columns, as well as the zeroes in the first and second columns, unchanged. Hence the net effect must just correspond to a different choice of the unit vector $\mathbf{n}$. Now multiplying V on the left by the 3×3 matrix $\Omega^\dagger$ (assumed real) will just rotate each column vector in the same way. Hence we end up with the three parameter effective matrix:

$$K = (\, \widehat{\mathbf{n}' \times \mathbf{a}'} \quad (\widehat{\mathbf{n}' \times \mathbf{a}'}) \times \mathbf{a}' \quad \hat{\mathbf{a}}'/\sqrt{2} \quad \hat{\mathbf{a}}'/\sqrt{2}\,)\,. \tag{5.3}$$

One may note that a change of variables to $\nu_3' = (\nu_3 + \nu_4)/\sqrt{2}$, $\nu_4' = (\nu_3 - \nu_4)/\sqrt{2}$ will convert the third column of K to $\hat{\mathbf{a}}'$ and the fourth to 0. In this basis K is just a real 3×3 orthogonal matrix which can be parametrized in one's favorite way. However, the basis used in (5.3) is preferable for discussing neutrino oscillations since, as we saw in Section 2, ν_3' and ν_4' are not mass eigenstates when one includes the ϵ^2 corrections.

The probability that a neutrino associated with charged lepton a oscillates in time t to one associated with charged lepton b is

$$I(a \to b) = \sum_{\alpha,\alpha'=1}^{4} K_{a\alpha} K_{b\alpha} K_{a\alpha'} K_{b\alpha'} \exp[i(E_{\alpha'} - E_\alpha)t], \tag{5.4}$$

where $E_{\alpha'} - E_\alpha \approx [m^2(\nu_{\alpha'}) - m^2(\nu_\alpha)]/2E$. New phenomena associated with $m(\nu_3) \neq m(\nu_4)$ are expected[7,9] in $\nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_\tau$ oscillations.

From Simpson's experiment[1,2] we may estimate $|K_{13}| = |K_{14}| \approx 0.1/\sqrt{2}$. $|K_{12}|$ should be obtained from the $\nu_e \leftrightarrow \nu_\mu$ oscillation explanation[10] of the solar neutrino flux deficit, while $|K_{23}| = |K_{24}| \lesssim 0.25$ from the lack of observation[23] of $\nu_\mu \to \nu_X$.

It seems worth emphasizing that a $(3,1)$ model is the minimal one needed to explain the solar flux and $\beta\beta_{0\nu}$ data. If the parallel generation structure is given up an interesting model may be constructed[7] with these fields alone.

6. Discussion

We have given in fairly explicit form the various neutrino and lepton mixing matrices which occur in the singular see-saw model characterized by a small hierarchy parameter ϵ. The difference in ϵ dependence from the non-singular see-saw case of some important amplitudes was studied. It was noted that the model seems reasonable for explaining the properties of Simpson's 17 keV neutrino. Of course, new Higgs particles which may enter in models of the present type might have additional astrophysical implications which must be studied.

The material here may be useful in treating a variety of models with different Higgs structures. The singularity in the heavy Majorana neutrino mass matrix may be easily achieved in a somewhat *ad hoc* way by imposition of suitable flavor dependent symmetries, generally involving the Higgs fields. In general the spontaneous breakdown of such symmetries will lead to Goldstone bosons different from the flavor singlet Majoron and the decay amplitude will be[24] of order ϵ^2 even for the non-singular see-saw. It is to be hoped that such symmetries would lead to a deeper understanding at the GUT or perhaps even superstring level. Indeed, the possibility of a singularity was suggested[16] by considering a GUT framework. Finally the origin of the particular value of the hierarchy parameter, $\epsilon \sim 10^{-7}$ is an interesting question; perhaps it suggests a "radiative" mechanism.[4,6,7]

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