Flux Tube Vibrations and the Excited Glue Spectroscopy

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As a step toward a fundamental understanding of hybrid states we demonstrate the close relationship between the relativistic string and lattice simulation results for excited glue states with two fixed color sources.

Because of the self-interacting nature of the gluon it is possible, even with fixed color ($Q\overline{Q}$) sources, to have an infinite set of excited glue states. The known mesons and baryons correspond to quark motion in the glue ground state configuration. For excited glue a vastly richer hadron spectrum of hybrid states then results, which can have important experimental consequences.

Before serious models of hybrid spectroscopy are attempted, it is important to investigate the central question of possible glue states with fixed color charges. From general symmetry considerations, the quantum glue can be labeled by the angular momentum $\Lambda = 0, 1, 2, \ldots$ along the $Q\overline{Q}$ axis (denoted by $\Sigma, \Pi, \Delta, \ldots$ in molecular notation), the CP quantum state $\eta_{\rm CP} = +1$ (denoted by g) and -1 (denoted by u), and finally for the $\Lambda = 0$ ($\Sigma^{\pm}_{g,u}$ states) a parity through a plane containing the $Q\overline{Q}$ axis. Although there is no way to directly access this excited glue spectrum experimentally, lattice simulation of QCD provides a very useful alternative. Recently a variety of glue states have been computed in this way over a wide range of $Q\overline{Q}$ separation[1]. It is our purpose here to show that this lattice data has a simple interpretation in terms of the relativistic vibrating flux tube[2].

It is well known that for large $Q\overline{Q}$ separations, the color field condenses into a string-like (or flux tube) configuration. It is then expected that quantized transverse vibrations of the flux tube provide a physical picture of excited glue states, at least for large $Q\overline{Q}$ separation[2]. The quantization of a vibrating string has an extensive literature[2]. We have recently pointed out that an isolated string can be quantized consistently in four dimensions, although an unknown parameter c appears due to an ordering ambiguity.

The Nambu-Goto action with fixed end boundary conditions may be quantized in $D = 4$ dimensions using the Gupta-Bleuler method[2]. The energy of the string with tension a and unit length r is

$$E(r) = \sqrt{(ar^2) + 2\pi a(N + c)} + C \qquad (1)$$

$$N = \sum_{n=1}^{\infty} n \left(N_n^+ + N_n^-\right) \qquad (2)$$

where N_n^+ is the phonon excitation number of level n and positive helicity. The parameter c

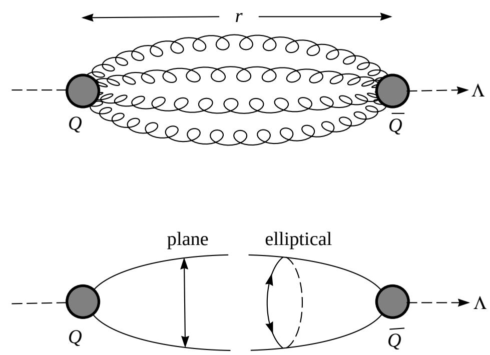

Figure 1. a) Glue states with fixed color sources. b) The flux tube picture of these states involves planar transverse vibration for excited Σ states or “jump-rope” states for Λ = 0.

is the ordering parameter referred to above, and we introduce the additive C associated with end renormalization. In the vibrating string picture the labels discussed earlier are

$$\Lambda = \sum_{n=1}^{\infty} \left(N_n^+ - N_n^-\right) \qquad (3)$$

$$\eta_{\rm CP} = (-)^N \qquad (4)$$

The glue state energy $E(r)$ is often called $V(r)$ since (in the spirit of Born and Oppenheimer) it can be thought of as a potential energy in which the (heavy) quarks slowly move.

We first consider the ground state ($N = 0$) potential describing the dynamics of ordinary mesons. In this case we must augment the expression (1) with a short-range color singlet interaction. The dimensionless potential is then

$$r_0 V(r/r_0) = \frac{-4\alpha_s/3}{(r/r_0)} + \sqrt{(ar_0^2)^2 (r/r_0)^2 + 2\pi (ar_0^2) c} + C r_0 , \qquad (5)$$

where r_0 is a hadronic scale distance[1]. If we compare this potential to the ground state (Σ_g) lattice simulation

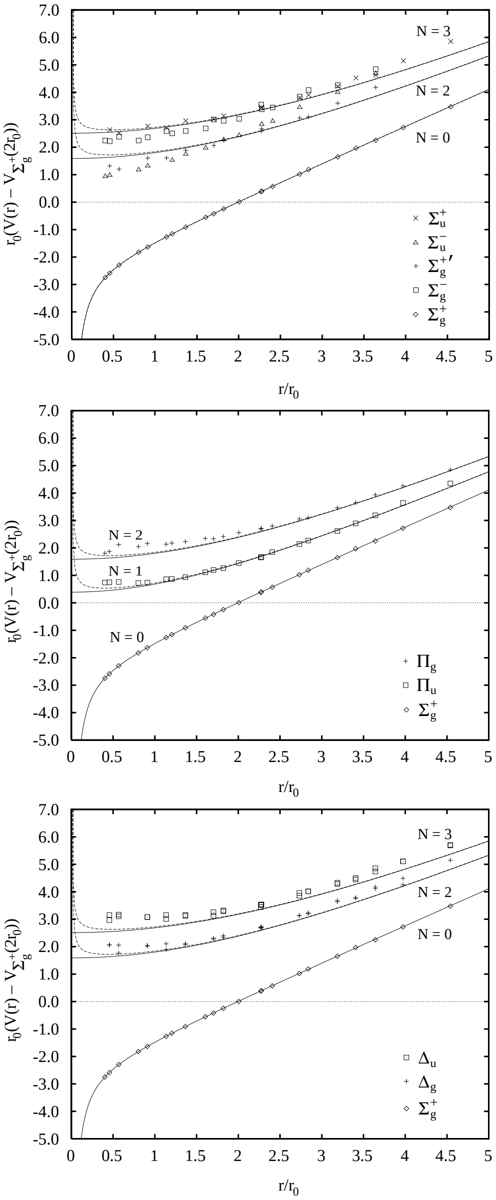

Figure 2. The Σ, Π, and Δ states ($\Lambda = 0, 1, 2$) respectively. The bottom line in each case is the best fit of Eq. (5) [using the parameters of (6)] to the ground state. The Σ_g glue state is given by lattice simulation of QCD[1]. The other solid curves are unique predictions of the quantized vibrating flux tube picture using Eq. (7) compared with lattice simulation[1].

shown in Fig. 2 we find the best fit occurs when

$$\begin{aligned} c &\quad 0\,, & r_0 C &\quad 2.51\,, \\ \alpha_s &\quad 0.234\,, & a r_0^2 &\quad 1.34\,. \end{aligned} \tag{6}$$

We note that with the ordering parameter $c = 0$, the potential (5) is the well-known “Cornell potential” and that it apparently exactly fits the simulation.

Once the above parameters are fixed, the excited glue energy predictions are unique:

$$r_0 V(r/r_0) = \sqrt{(a r_0^2)^2 (r/r_0)^2 + 2\pi (a r_0^2) N} + r_0 C\,. \tag{7}$$

The predictions and lattice simulation data[1] are shown in Fig. 2. There is only one lowest excited glue state ($N = 1$) and it corresponds to $\eta_{CP} = -1$ and $\Lambda = 1$, denoted by Π_u in molecular notation. For $N \geq 2$ there are Σ, Π, and Δ... states predicted. As seen in Fig. 2, the vibrating flux tube expectations work remarkably well.

In conclusion, we have shown that the features of the simulated glue spectroscopy with fixed $Q\overline{Q}$ sources[1] can be understood in terms of a quantized transversely vibrating flux tube. An isolated string/tube can be consistently quantized in four dimensions[2]. The agreement between the tube predictions and lattice QCD indicates that the vibrating flux tube picture is valid.

REFERENCES

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2. Theodore J. Allen, M.G. Olsson, and Siniša Veseli, *Excited Glue and the Vibrating Flux Tube*, Univ. of Wisconsin-Madison report MADPH-98-1056 [hep-ph/9804452] (Phys. Lett. B, in press). References to previous work are given therein.