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Axionic Black Holes from Massive Axions

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Abstract

A black hole may carry axionic charge in a theory which has gravity coupled to a massless two-form Kalb-Ramond field. We show that this effect persists if the axion has a topological mass term coupling the Kalb-Ramond potential to a $U(1)$ gauge field. Such mass terms arise in the low-energy effective theory of the string.

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I. Introduction

The stationary black hole solutions to Einstein's equations in vacuo [1] are uniquely characterized by the mass M and angular momentum J [2]. Generally one expects that the gravitational field must be coupled to a massless field for a black hole solution to acquire another parameter or charge. For the coupled Einstein-Maxwell equations there are two additional parameters, the electric charge e and the magnetic charge μ [3] and again there is a uniqueness theorem [4]. A great deal of the interest and importance of these classical solutions derives from their uniqueness, since they determine the final state of gravitational collapse of arbitrarily complicated states of matter with sufficiently high mass. Charges, such as baryon number, which do not appear to be coupled to a massless field generically decouple as they approach the event horizon and cease to be observable [5].

Recently it was found that black holes may also carry axionic charge [6]. It was assumed in [6] that the axion field was exactly massless and is the physical mode of a two-form potential, called the Kalb-Ramond (KR) field. KR fields arise in string theory as they naturally couple to the area element of the two-dimensional worldsheet [7]. They also arise in supergravity models, and in cosmic strings arising from the spontaneous breaking of anomalous $U(1)$ symmetries [8]. In particular, KR fields are contained in string theories compactified to four dimensions. A variety of physical mechanisms, however, such as those associated with instantons, wormholes and the generic compactification to four dimensions, are expected to generate masses for axions. We will start by reviewing the massless case and its assumptions and then proceed to the consideration of the massive case.

II. The Massless Case

We consider a two-form $B_{\mu\nu}$ in four dimensions which interacts with gravity but is massless. The action for this field is

$$S = \tfrac{1}{2} \int d^4x (H \wedge {}^*H). \tag{2.1}$$

The field strength is given by $H = dB$. The field equations which follow from this action,

$$d\,{}^*H = 0, \tag{2.2}$$

can be solved in a simply-connected space by introducing a scalar ϕ:

$${}^*H = d\phi. \tag{2.3}$$

The Bianchi identity for H, $dH = 0$, becomes the field equation for ϕ,

$$\Box\phi = 0. \tag{2.4}$$

The field ϕ has a global Peccei-Quinn $U(1)$ symmetry,

$$\phi \rightarrow \phi + a, \tag{2.5}$$

where a is any constant. It should be noted that this correspondence is only valid for the free case, since off-shell these theories have different numbers of degrees of freedom. One can most easily count the degrees of freedom for the two-form using the canonical approach. On the phase space consisting of $B_{\mu\nu}$ and its canonically conjugate momentum $\Pi^{\mu\nu}$ there are first-class constraints, analogous to those of electromagnetism:

$$\begin{aligned} \Pi^{0i}(\mathbf{x}) &\approx 0, \\ \partial_i \Pi^{ij}(\mathbf{x}) &\approx 0. \end{aligned} \tag{2.6}$$

In addition there is a reducibility condition on the last constraint of (2.6),

$$\partial_i \partial_j \Pi^{ij}(\mathbf{x}) \equiv 0. \tag{2.7}$$

Thus there are exactly five first-class constraints on $B_{\mu\nu}(\mathbf{x})$ and its momentum $\Pi^{\mu\nu}(\mathbf{x})$. This leaves one degree of freedom in the configuration space. When the

KR field becomes massive, there are two additional degrees of freedom. In the momentum representation, the associated modes are of the form

$$B_{ij}(\mathbf{k}) = \lambda_{[i} k_{j]}, \tag{2.8}$$

where each three-vector $\vec{\lambda}$ is orthogonal to the three-momentum $\vec{k}$. The same argument also yields the correct degrees of freedom for both a massless and massive gauge field. The KR symmetries (needed to keep the massless KR field equivalent to a scalar particle) will restrict the types of terms which can be induced by renormalization or instanton effects. If the definition of $H_{\mu\nu\rho}$ is modified to include the Chern-Simons forms of the gauge and gravitational fields, as is the case in the low-energy limit of superstring theory, then ϕ has the correct coupling to gauge and gravitational degrees of freedom to be an axion. Here, however, we do not consider the Chern-Simons terms.

The axionic charge in a spatial volume V was defined in [6] to be

$$q_{\text{axionic}} = \int_V H = \oint_{\partial V} B. \tag{2.9}$$

It is not difficult to prove [6] that the coupled Einstein-axion field equations admit the solution

$$\begin{aligned} ds^2 &= -(1 - \tfrac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2, \\ B &= q\varpi, \end{aligned} \tag{2.10}$$

with ϖ a harmonic generator of the second cohomology of the spacetime. In this section we give an alternative proof of uniqueness (up to diffeomorphisms and KR gauge transformations) which generalizes to the massive case. We assume staticity, asymptotic flatness and regularity of the horizons. By staticity we mean that there exists a time-like Killing vector which Lie derives the fields and is also hypersurface orthogonal. (We will call a spacetime stationary if there exists a time-like Killing vector which is not hypersurface orthogonal.) Asymptotic flatness means that the spacetime metric approaches the Minkowski metric at least as fast as $\frac{1}{r}$. Finally, regularity of the horizons is the requirement that all fields remain finite at all of the spacetime horizons. In the following we will denote the hypersurface orthogonal Killing vector by ξ_μ. If we define the projection operator

$$\Pi^\mu_\nu = \delta^\mu_\nu + \lambda^{-2}\xi^\mu\xi_\nu, \tag{2.11}$$

where λ is the length of the Killing vector ξ_μ, $\lambda^2 = -\xi_\mu\xi^\mu$, the Frobenius condition for hypersurface orthogonality can be expressed as

$$\Pi^\mu_\nu\Pi^\sigma_\tau\nabla_{[\mu}\xi_{\sigma]} = 0. \tag{2.12}$$

The uniqueness proof in both the massless and the massive case hinges on a simple relation between the projection of the spacetime divergence of a p-form Ω, which is Lie derived by the Killing vector ξ, and the hypersurface divergence of its hypersurface projection, ω. This divergence relation is

$$D_\alpha(\lambda\omega^{\alpha\mu_1\cdots\mu_{p-1}}) = \lambda\Pi^{\mu_1}_{\nu_1}\cdots\Pi^{\mu_{p-1}}_{\nu_{p-1}}\nabla_\beta\Omega^{\beta\nu_1\cdots\nu_{p-1}}, \tag{2.13}$$

where D_α is the induced connection on the hypersurface. The first step in the proof of uniqueness is to consider both the field strength $H_{\mu\nu\rho}$ and its spacetime dual ${}^*H_\mu$ and to project both of them down to the hypersurface. We shall denote the projected fields by $h_{\mu\nu\rho}$ and d_μ respectively. Using the divergence relation (2.13), the equation of motion and the Bianchi identity, we have

$$\begin{aligned} D_\mu(\lambda h^{\mu\nu\rho}) &\propto \lambda\nabla_\mu H^{\mu\nu\rho} = 0, \\ D_\mu(\lambda d^\mu) &= \lambda\nabla_\mu {}^*H^\mu = 0. \end{aligned} \tag{2.14}$$

We assume that the field strengths fall off at infinity (where the metric is flat) at least as fast as $\frac{1}{r^2}$ and that we can find potentials ϕ and $b_{\mu\nu}$ which fall off to zero as fast as $\frac{1}{r}$ and satisfy

$$\begin{aligned} d &= D\phi, \\ h &= Db. \end{aligned} \tag{2.15}$$

Then we multiply (2.14) by the appropriate potentials and integrate by parts to obtain

$$\begin{aligned} 0 &= \int\limits_V b_{\nu\rho} D_\mu(\lambda h^{\mu\nu\rho}) = -\int\limits_V \lambda h^2 \Rightarrow h = 0, \\ 0 &= \int\limits_V \phi D_\mu(\lambda d^\mu) = -\int\limits_V \lambda d^2 \Rightarrow d = 0. \end{aligned} \tag{2.16}$$

The region V is that region of a hypersurface which is outside of all horizons and extends out to infinity. On the horizons, the Killing vector ξ_μ becomes null, so λ goes to zero. As we are assuming that the fields are finite at the horizons and vanish sufficiently rapidly at infinity, the surface integrals resulting from the integration by parts must vanish. Since the projections h and d are both zero on any region V as above, and since these regions foliate spacetime, it follows that the field strength $H_{\mu\nu\rho}$ vanishes in all regions of spacetime outside of the horizons and that the spacetime is a vacuum spacetime. Israel's vacuum uniqueness theorem [9] takes care of the proof of the uniqueness of the metric.

III. The Massive Axion

In theories where the axion is described by a two form potential, such as superstring theories, there may be a mass term for this potential and, indeed, we expect that generically such mass terms should be present in the low energy limit of the string theory. What we show below is that these terms do not spoil the existence and uniqueness of the solutions (2.10) to the coupled KR-Einstein field equations. This is so primarily because the existence of the KR symmetry greatly restricts the form of the interactions in which $B_{\mu\nu}$ may partake. The mass term which we investigate is the so-called topological mass term. We consider some generic $U(1)$ gauge field A_μ with which $B_{\mu\nu}$ may interact. In some sense, this is the natural field to consider adding to the action, since when we are done, the KR field will eat the two physical modes of the gauge field necessary to become massive. Alternatively, one may consider that the gauge field will eat the single degree of freedom of the KR field to become massive itself.

Consider, then, the free action for $B_{\mu\nu}$ and A_μ,

$$S_{\text{kin}} = \tfrac{1}{2}\int d^4x[H \wedge {}^*H + F \wedge {}^*F], \tag{3.1}$$

to which we add the topological mass term

$$S_{\text{mass}} = m\int d^4x[B \wedge F]. \tag{3.2}$$

As usual, $H = dB$ and $F = dA$. This topological mass term is gauge invariant since it transforms into a total derivative,

$$\delta S_{\text{mass}} = m\int d^4x[d\Lambda \wedge F], \tag{3.3}$$

under a Kalb-Ramond gauge transformation $\delta B = d\Lambda$. This term is, of course, invariant under $U(1)$ gauge transformations as well.

The equations of motion which follow from the action (3.1) and (3.2) are

$$\begin{aligned} d\,{}^*H &= mF, \\ d\,{}^*F &= -mH. \end{aligned} \tag{3.4}$$

Applying $d*$ to both equations and substituting, we obtain the massive Klein-Gordon equations

$$\begin{aligned} (\Box - m^2)H &= 0, \\ (\Box - m^2)F &= 0. \end{aligned} \tag{3.5}$$

Obviously, we are describing massive degrees of freedom. If we assume that the spacetime is simply connected, we may solve the first equation of (3.4) by introducing a scalar field ϕ,

$$^{*}H = d\phi + mA. \tag{3.6}$$

One can see that the axion ϕ isn't gauge invariant. In fact, we can force the axion to be the longitudinal degree of freedom by choosing the gauge

$$m\phi + \partial_\mu A^\mu = 0, \tag{3.7}$$

and in doing, obtain the Klein-Gordon equation for the $U(1)$ field.

$$(\Box - m^2)A_\mu = 0. \tag{3.8}$$

Alternatively, we may solve the second equation of motion in (3.4) using a dual potential G_μ for the field strength F:

$$^{*}F = dG - mB + p\varpi. \tag{3.9}$$

Choosing the gauge

$$\mathrm{div}B + mG = 0, \tag{3.10}$$

and making the large gauge transformation $B' = B + \frac{p}{m}\varpi$, we find the equation of motion for the new B',

$$(\Box - m^2)B' = 0. \tag{3.11}$$

Thus B becomes massive by swallowing the $U(1)$ gauge field. Again, there is a single unique static solution of the massive Kalb-Ramond- Einstein-Maxwell field equations. To prove this, we must consider the projections of both $F^{\mu\nu}$ and $H^{\mu\nu\rho}$ and their spacetime duals. Let us denote these projections by

$$\begin{aligned} F^{\mu\nu} &\to f^{\mu\nu}, \\ {}^*F^{\mu\nu} &\to e^{\mu\nu}, \\ H^{\mu\nu\rho} &\to h^{\mu\nu\rho}, \\ {}^*H^{\mu} &\to d^{\mu}. \end{aligned} \tag{3.12}$$

We compute

$$\begin{aligned} D_\rho(\lambda h^{\mu\nu\rho}) &= \lambda \Pi^\mu_\alpha \Pi^\nu_\beta \nabla_\gamma H^{\alpha\beta\gamma} = -\lambda m\, e^{\mu\nu}, \\ D_\mu(\lambda f^{\mu\nu}) &= \lambda m\, d^\nu, \end{aligned} \tag{3.13}$$

multiply each by the appropriate field strength, and integrate over the region between the horizon and spatial infinity.

$$\begin{aligned} \int_V e_{\mu\nu}[D_\rho(\lambda h^{\mu\nu\rho}) + \lambda m\, e^{\mu\nu}] = 0, \\ \int_V d_\nu[D_\mu(\lambda f^{\mu\nu}) - \lambda m\, d^\nu] = 0. \end{aligned} \tag{3.14}$$

To finish the proof we integrate by parts using the equations of motion, and argue that the boundary terms vanish by the assumptions of asymptotic flatness and regularity of the horizon.

$$\begin{aligned} \int_V \lambda m (h_{\mu\nu\rho} h^{\mu\nu\rho} + e_{\mu\nu} e^{\mu\nu}) = 0, \\ \int_V \lambda m (d_\nu d^\nu + f_{\mu\nu} f^{\mu\nu}) = 0. \end{aligned} \tag{3.15}$$

Thus, both $F_{\mu\nu}$ and $H_{\mu\nu\rho}$ vanish throughout the region V, since the metric is positive definite on the hypersurface. This holds for any of the hypersurfaces defined by orthogonality to ξ, which foliate spacetime, from which it follows that $F_{\mu\nu}$ and $H_{\mu\nu\rho}$ vanish everywhere in spacetime. Therefore, the axionically charged black hole solution (2.10) holds and is unique for the topologically massive case as well. Note that the black hole is not allowed to carry any $U(1)$ charge for the field which couples to the axion, since the integral of F over any closed surface around the black hole is zero.

IV. Stückelberg Lagrangian

There is another Lagrangian which will yield a massive axion, while still allowing a solution with an axionic charge q. The gauge invariance must now be put in by hand with the aid of a Stückelberg-type mechanism. This involves introducing a vector field χ, which is similar to the field A of the previous section, but has a fundamentally different coupling with the KR field, B. The Lagrangian is written as

$$\mathcal{L} = \tfrac{1}{2} H \wedge {}^*H + \tfrac{1}{2} m^2 (B - q\varpi - d\chi) \wedge {}^*(B - q\varpi - d\chi). \tag{4.1}$$

Again, the quantity ϖ is a harmonic generator of the second cohomology of the spacetime background we are considering. This Lagrangian still has the Kalb-Ramond symmetry $B \to B + d\Lambda$, $\chi \to \chi + \Lambda$. The equations which follow from this action are

$$\begin{aligned} d\,{}^*H - m^2\,{}^*(B - q\varpi - d\chi) &= 0, \\ d\,{}^*(B - q\varpi - d\chi) &= 0. \end{aligned} \tag{4.2}$$

We may choose a gauge in which χ is set to zero. The relevant equations are then

$$\begin{aligned} \mathrm{div} H &= m^2 (B - q\varpi), \\ \mathrm{div}\,{}^*(B - q\varpi) &= {}^*H. \end{aligned} \tag{4.3}$$

The quantities whose divergences appear on the left hand side are assumed to be Lie-derived by the Killing vector ξ_μ, so we can apply the divergence relation to them. After some short algebra, we again obtain equalities which imply that $H_{\mu\nu\rho} = 0$, and $B_{\mu\nu} = q\varpi_{\mu\nu}$.

V. Discussion

We have demonstrated, under fairly mild assumptions, that an axion which gets a mass through either a topological or Stückelberg mechanism can give rise to an axionic charge on a black hole. We have not investigated axions coming specifically from the low-energy limit of string theory, as we have neglected the Chern-Simons terms in the KR field strength. Neither have we analyzed the complicated terms in the string-theory effective action of the form $\int B \wedge X_8$ necessary for the Green-Schwarz anomaly cancellation. We shall just point out that one of these terms, $\int B \wedge \mathrm{tr}(F^4)$, generally leads to topological mass terms of the form $m \int B \wedge F$ upon compactification to four dimensions [10,11]. We would like to argue that mass terms of the Stückelberg type (4.1) are not realistic from the point of view of string theory, simply because there are no obvious massless vector fields with the right couplings in the low-energy limit necessary to play the role of Stückelberg fields. Such terms are not ruled out, however, if one's only criterion is gauge invariance.

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