MATH 375.1

Class 2: Selected Answers

1. The table for $(\mathbf{Z}_5^*, \odot)$ is on the left below. It is a commutative group ($e = 1$).

$\odot$ mod 5	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

$\odot$ mod 4	1	2	3
1	1	2	3
2	2	0	2
3	3	2	1

2. **a)** The table for $(\mathbf{Z}_4^*, \odot)$ is on the right above. It is commutative. Again $e = 1$ is the identity, but it is not closed since $2 \odot 2 = 0$, and 2 has no inverse.

b) $(\mathbf{Z}_6^*, \odot)$ will not be a group because it is not closed: $2 \odot 3 = 0$. Though $e = 1$, neither 2 nor 4 will have inverses.

3. Let $z = 2 + 5i$ and $w = 4 + 3i$. Calculate the following sums and products:

a) $z + w = 6 + 8i$ **b)** $z - w = -2 + 2i$ **c)** $zw = -7 + 26i$ **d)** $iz = -5 + 2i$

4. **a)** The the set $G = \{1, i, -1, -i\}$ of complex numbers undercomplex multiplication is a group with $e = 1$ and it is commutative.

$(G, \times)$	1	i	-1	$-i$
1	1	i	-1	$-i$
i	i	-1	$-i$	1
-1	-1	$-i$	1	i
$-i$	$-i$	1	i	-1

5. **a)** Find all numbers less than $n = 30$ that are relatively prime to 30. That is, find k so that $\gcd(30, k) = 1$. $k = 1$, 7, 11, 13, 17, 19, 23, and 29.

b) Check that $81 \cdot 27 = 3 \bmod 12$.

c) Find $\gcd(8767, 2178)$.

$$
\begin{aligned}
8767 &= 2178 \cdot 4 + 55 \\
2178 &= 55 \cdot 39 + 33 \\
55 &= 33 \cdot 1 + 22 \\
33 &= 22 \cdot 1 + 11 \\
22 &= 11 \cdot 2 + 0
\end{aligned}
$$

So $\gcd(8767, 2178) = 11$.