MATH 375.1

Class 1: Selected Answers

1. **a)** There are four rigid motions of a rectangle: r_0, r_{180}, v, and h, where v is the reflection across the vertical bisector of the rectangle and h is the reflection across the horizontal bisector.

b) Check that

$*$	r_0	r_{180}	v	h
r_0	r_0	r_{180}	v	h
r_{180}	r_{180}	r_0	h	v
v	v	h	r_0	r_{180}
h	h	v	r_{180}	r_0

c) Each element appears once in each row and each column.

d) $r0$ is the identity element. Each element is its own inverse.

e) It is closed and commutative. It is a group (check associativity).

2. The Cayley Table for $(\mathbf{Z}_4, \oplus)$ is not the same as the one above. In the $\mathbf{Z}_4$ table, not every element is its own inverse, so the identity does not always appear on the diagonal.

3. $(\mathbf{Z}_5, \oplus)$ is also a commutative group. See the table on the left below.

$\oplus$ mod 5	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$\odot$ mod 5	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

4. The table for $(\mathbf{Z}_5, \odot)$ is on the right above. It is commutative. It is not a group. 1 is the identity, but 0 has no inverse. Note that 0 is a problem in that it appears too often in its row and column.

5. The Cayley table for the set of motions of a square is:

$*$	r_0	r_{90}	r_{180}	r_{270}	h	v	d	d'
r_0	r_0	r_{90}	r_{180}	r_{270}	h	v	d	d'
r_{90}	r_{90}	r_{180}	r_{270}	r_0	d	d'	v	h
r_{180}	r_{180}	r_{270}	r_0	r_{90}	v	h	d'	d
r_{270}	r_{270}	r_0	r_{90}	r_{180}	d'	d	h	v
h	h	d'	v	d	r_0	r_{180}	r_{270}	r_{90}
v	v	d	h	d'	r_{180}	r_0	r_{90}	r_{270}
d	d	h	d'	v	r_{90}	r_{270}	r_0	r_{180}
d'	d'	v	d	h	r_{270}	r_{90}	r_{180}	r_0

This is the group D_4, the dihedral group of order 8. It is not commutative (not the table is not symmetric about the main diagonal). (h is the horizontal reflection, v the vertical, d is the reflection across the main diagonal, and d' is the reflection across the off diagonal.)