Standard 1—Analysis, Inquiry, and Design

Intermediate

Mathematical Analysis

1. Abstraction and symbolic representation are used to communicate mathematically.

Students:

- **extend mathematical notation and symbolism to include variables and algebraic expressions in order to describe and compare quantities and express mathematical relationships.**

2. Deductive and inductive reasoning are used to reach mathematical conclusions.

Students:

- **use inductive reasoning to construct, evaluate, and validate conjectures and arguments, recognizing that patterns and relationships can assist in explaining and extending mathematical phenomena.**

This is evident, for example, when students:

▲ predict the next triangular number by examining the pattern 1, 3, 6, 10, ❒.

3. Critical thinking skills are used in the solution of mathematical problems.

Students:

- **apply mathematical knowledge to solve real-world problems and problems that arise from the investigation of mathematical ideas, using representations such as pictures, charts, and tables.**

Key ideas are identified by numbers (1).
Performance indicators are identified by bullets (•).
Sample tasks are identified by triangles (▲).

Scientific Inquiry

1. The central purpose of scientific inquiry is to develop explanations of natural phenomena in a continuing, creative process.

Students:

- **formulate questions independently with the aid of references appropriate for guiding the search for explanations of everyday observations.**
- **construct explanations independently for natural phenomena, especially by proposing preliminary visual models of phenomena.**
- **represent, present, and defend their proposed explanations of everyday observations so that they can be understood and assessed by others.**
- **seek to clarify, to assess critically, and to reconcile with their own thinking the ideas presented by others, including peers, teachers, authors, and scientists.**

This is evident, for example, when students:

▲ *After being shown the disparity between the amount of solid waste which is recycled and which could be recycled,** students working in small groups are asked to explain why this disparity exists. They develop a set of possible explanations and to select one for intensive study. After their explanation is critiqued by other groups, it is refined and submitted for assessment. The explanation is rated on clarity, plausibility, and appropriateness for intensive study using research methods.

2. Beyond the use of reasoning and consensus, scientific inquiry involves the testing of proposed explanations involving the use of conventional techniques and procedures and usually requiring considerable ingenuity.

Students:

- **use conventional techniques and those of their own design to make further observations and refine their explanations, guided by a need for more information.**
- **develop, present, and defend formal research proposals for testing their own explanations of common phenomena, including ways of obtaining needed observations and ways of conducting simple controlled experiments.**
- **carry out their research proposals, recording observations and measurements (e.g., lab notes, audio tape, computer disk, video tape) to help assess the explanation.**

This is evident, for example, when students:

▲ develop a research plan for studying the accuracy of their explanation of the *disparity between the amount of solid waste that is recycled and that could be recycled.** After their tentative plan is critiqued, they refine it and submit it for assessment. The research proposal is rated on clarity, feasibility and soundness as a method of studying the explanations' accuracy. They carry out the plan, with teacher suggested modifications. This work is rated by the teacher while it is in progress.

Students will use mathematical analysis, scientific inquiry, and engineering design, as appropriate, to pose questions, seek answers, and develop solutions.

3. The observations made while testing proposed explanations, when analyzed using conventional and invented methods, provide new insights into phenomena.

Students:

- **design charts, tables, graphs and other representations of observations in conventional and creative ways to help them address their research question or hypothesis.**
- **interpret the organized data to answer the research question or hypothesis and to gain insight into the problem.**
- **modify their personal understanding of phenomena based on evaluation of their hypothesis.**

This is evident, for example, when students:

▲ carry out their plan making appropriate observations and measurements. They analyze the data, reach conclusions regarding their explanation of the *disparity between the amount of solid waste which is recycled and which could be recycled.**, and prepare a tentative report which is critiqued by other groups, refined, and submitted for assessment. The report is rated on clarity, quality of presentation of data and analyses, and soundness of conclusions.

Engineering Design

1. Engineering design is an iterative process involving modeling and optimization finding the best solution within given constraints which is used to develop technological solutions to problems within given constraints.

Students engage in the following steps in a design process:

- **identify needs and opportunities for technical solutions from an investigation of situations of general or social interest.**
- **locate and utilize a range of printed, electronic, and human information resources to obtain ideas.**
- **consider constraints and generate several ideas for alternative solutions, using group and individual ideation techniques (group discussion, brainstorming, forced connections, role play); defer judgment until a number of ideas have been generated; evaluate (critique) ideas; and explain why the chosen solution is optimal.**
- **develop plans, including drawings with measurements and details of construction, and construct a model of the solution, exhibiting a degree of craftsmanship.**
- **in a group setting, test their solution against design specifications, present and evaluate results, describe how the solution might have been modified for different or better results, and discuss tradeoffs that might have to be made.**

This is evident, for example, when students:

▲ reflect on the need for alternative growing systems in desert environments and design and model a hydroponic greenhouse for growing vegetables without soil.

▲ brainstorm and evaluate alternative ideas for an adaptive device that will make life easier for a person with a disability, such as a device to pick up objects from the floor.

▲ design a model vehicle (with a safety belt restraint system and crush zones to absorb impact) to carry a raw egg as a passenger down a ramp and into a barrier without damage to the egg.

▲ assess the performance of a solution against various design criteria, enter the scores on a spreadsheet, and see how varying the solution might have affected total score.

* A variety of content-specific items can be substituted for the italicized text