1. Abstraction and symbolic representation are used to communicate mathematically.

Students:
- use algebraic and geometric representations to describe and compare data.

2. Deductive and inductive reasoning are used to reach mathematical conclusions.

Students:
- use deductive reasoning to construct and evaluate conjectures and arguments, recognizing that patterns and relationships in mathematics assist them in arriving at these conjectures and arguments.

3. Critical thinking skills are used in the solution of mathematical problems.

Students:
- apply algebraic and geometric concepts and skills to the solution of problems.

1. The central purpose of scientific inquiry is to develop explanations of natural phenomena in a continuing, creative process.

Students:
- elaborate on basic scientific and personal explanations of natural phenomena, and develop extended visual models and mathematical formulations to represent their thinking.
- hone ideas through reasoning, library research, and discussion with others, including experts.
- work toward reconciling competing explanations; clarifying points of agreement and disagreement.
- coordinate explanations at different levels of scale, points of focus, and degrees of complexity and specificity and recognize the need for such alternative representations of the natural world.

This is evident, for example, when students:
- in small groups, are asked to explain why a cactus plant requires much less water to survive than many other plants.* They are asked to develop, through research, a set of explanations for the differences and to select at least one for study. After the proposed explanation is critiqued by others, they refine it by formulating a hypothesis which is rated on clarity, plausibility, and researchability.

2. Beyond the use of reasoning and consensus, scientific inquiry involves the testing of proposed explanations involving the use of conventional techniques and procedures and usually requiring considerable ingenuity.

Students:
- devise ways of making observations to test proposed explanations.
- refine their research ideas through library investigations, including electronic information retrieval and reviews of the literature, and through peer feedback obtained from review and discussion.
- develop and present proposals including formal hypotheses to test their explanations, i.e., they predict what should be observed under specified conditions if the explanation is true.
- carry out their research plan for testing explanations, including selecting and developing techniques, acquiring and building apparatus, and recording observations as necessary.

This is evident, for example, when students:
- develop, through research, a proposal to test their hypothesis of why a cactus plant requires much less water to survive than many other plants.* After their proposal is critiqued, it is refined and submitted for assessment by a panel of students. The proposal is rated on clarity, appropriateness, and feasibility. Upon approval, students complete the research. Progress is rated holistically by the teacher.
3. The observations made while testing proposed explanations, when analyzed using conventional and invented methods, provide new insights into phenomena.

Students:
- use various means of representing and organizing observations (e.g., diagrams, tables, charts, graphs, equations, matrices) and insightfully interpret the organized data.
- apply statistical analysis techniques when appropriate to test if chance alone explains the result.
- assess correspondence between the predicted result contained in the hypothesis and the actual result and reach a conclusion as to whether or not the explanation on which the prediction was based is supported.
- based on the results of the test and through public discussion, they revise the explanation and contemplate additional research.
- develop a written report for public scrutiny that describes their proposed explanation, including a literature review, the research they carried out, its result, and suggestions for further research.

This is evident, for example, when students:
- carry out a research plan, including keeping a lab book, to test their hypothesis of why a cactus plant requires much less water to survive than many other plants.* After completion, a paper is presented describing the research. Based on the class critique, the paper is rewritten and submitted with the lab book for separate assessment or as part of a portfolio of their science work. It is rated for clarity, thoroughness, soundness of conclusions, and quality of integration with existing literature.

Engineering Design

1. Engineering design is an iterative process involving modeling and optimization finding the best solution within given constraints which is used to develop technological solutions to problems within given constraints.

Students engage in the following steps in a design process:
- initiate and carry out a thorough investigation of an unfamiliar situation and identify needs and opportunities for technological invention or innovation.
- identify, locate, and use a wide range of information resources, and document through notes and sketches how findings relate to the problem.
- generate creative solutions, break ideas into significant functional elements, and explore possible refinements; predict possible outcomes using mathematical and functional modeling techniques; choose the optimal solution to the problem, clearly documenting ideas against design criteria and constraints; and explain how human understands, economics, ergonomics, and environmental considerations have influenced the solution.
- develop work schedules and working plans which include optimal use and cost of materials, processes, time, and expertise; construct a model of the solution, incorporating developmental modifications while working to a high degree of quality (craftsmanship).
- devise a test of the solution according to the design criteria and perform the test; record, portray, and logically evaluate performance test results through quantitative, graphic, and verbal means. Use a variety of creative verbal and graphic techniques effectively and persuasively to present conclusions, predict impacts and new problems, and suggest and pursue modifications.

This is evident, for example, when students:
- search the Internet for world wide web sites dealing with renewable energy and sustainable living and research the development and design of an energy efficient home.
- develop plans, diagrams, and working drawings for the construction of a computer-controlled marble sorting system that simulates how parts on an assembly line are sorted by color.
- design and model a portable emergency shelter that could be heated by a person’s body to a life-sustaining temperature when the outside temperature is 20° F.

* A variety of content-specific items can be substituted for the italicized text