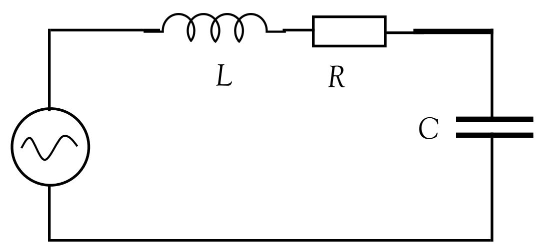
Lab 2

RLC Circuits: The Driven, Damped, Oscillator

Refer to Smith sections 3.2-3.5 and 4.1-4.3.

You are already familiar with the basics of simple harmonic motion in the context of mechanical oscillators. The goal for today is to explore some of the subtler details of resonance phenomena in the context of the electrical analog to that familiar mechanical system, namely the *RLC* circuit. While we hope that the observations will largely confirm the results of our calculations, we will also be alert to the possibility that real circuits, built out of real components, may not be fully described by the idealized components appearing in our calculations.



The series *RLC* oscillator is governed by the same equation as a driven mechanical oscillator:

$$m\ddot{x} + b\dot{x} + kx = F(t) \quad \text{Mechanical Oscillator}$$
$$L\ddot{q} + R\dot{q} + \tfrac{1}{C}q = V(t) \quad \text{Electrical Oscillator}$$

where q is the charge on the capacitor. We can draw the following correspondences:

position (x) $\Leftrightarrow$ charge on the capacitor (q)
velocity $\left(\dot{x}\right)$ $\Leftrightarrow$ current (I) (technically through C, but in this circuit I is the same everywhere)
mass m $\Leftrightarrow$ inductance L
spring constant (k) $\Leftrightarrow$ inverse of capacitance ($1/C$)
damping constant (b) $\Leftrightarrow$ resistance (R)

The order of the components in the circuit does not matter as long as the R, L and C are connected in series; you can move the components around as convenient. Recall the importance of having all of your 'grounds' connected to a common circuit point – you may need to move components around in order to both measure what you want and have only one point in the circuit grounded.

In this lab you will be looking at your system in two ways. First (and only briefly), you will look at the response of the system when it is "plucked," i.e. it is pulled away from equilibrium and then allowed to respond freely. This is similar to what you did last week in measuring the decay/rise of the voltage in an *RC* circuit, the novel feature here is that the *RLC* circuit oscillates while decaying. The second type of measurement, and the main focus of the lab, is to look at the response of your system when it is driven with a sinusoidal signal. At the end you will do a slight variation on this theme and look briefly at the response of the circuit when it is driven periodically but with a square wave instead of a sine wave.

Technical details about circuit components: The labeling convention on the drawers containing the components is that the top item on the label is in the front section of the drawer, the bottom item is in the back. To start, locate the inductors – you will need both 1 mH ("milliHenry") and 10 mH. You will also need capacitors, 0.1 nF, 1.0 nF and 10 nF. The tradition in marking capacitors is to use pF or μF hence a 1 nF capacitor will be labeled as either "0.001μF" or "1000 pF", while a 10 nF capacitor will be designated "0.01 μF" or "10,000 pF." The actual markings on the circuit elements are not standardized. Some small capacitors just have the value in pF. The 1nF capacitor is marked 102. This means $10\text{x}10^2$ pF = 1,000 pF (which is 1 nF or 0.001 μF). Likewise the 1 mH inductor is marked 105 meaning $10\text{x}10^5$ nH = 1 mH. If in doubt you can use the fancy "R-L-C Digibridge" meter to check component values!

Measure the DC resistance of the 1 mH inductor using a multimeter. We will want to refer back to this measurement later.

1. Set up the circuit. Set up your signal generator as we did last week with an extra resistor positioned so as to reduce the output impedance of the signal generator. A resistor of around 10 ohms should be sufficient. This resistor is not shown explicitly any of the diagrams! It is considered part of the signal generator.

   Set up the circuit above with *L*= 1 mH, C= 1nF, *R*=10 Ω. Connect the scope so that you can monitor the voltage out of the signal generator on CH 1 and the voltage across the resistor, V_R, on CH 2. Since the voltage across the resistor is, up to a constant, just the current through the circuit, we will refer to it as that. The current is the analog to the velocity in a mechanical oscillator. Make sure you have arranged the components and the 'scope so that only one point in the circuit is grounded!

2. Plucked behavior (free, damped oscillations). Start by measuring the "plucked" behavior of a lightly damped circuit (i.e. by driving the circuit with a long period square wave so that the circuit response has time do die down to essentially zero in between "plucks."). In the text (section 3.2) it is shown that the amplitude decays like $e^{-\frac{\gamma}{2}t}$ where $\gamma = R/L$. Measure both the oscillation frequency and the damping (*1/e*) time. Do your measurements agree with what you expect from the component values? (If you find that the damping time does not agree with the component values: hold that thought. We will pursue this issue shortly).

3. Changing the damping. As discussed in Smith section 3.5 an oscillator is "critically damped" when $Q = \omega_0/\gamma = 1/2$. If you increase the damping in the system enough to reach this value, the motion of the plucked oscillator goes from being an oscillating function to just an exponential decay. Try this with your circuit, increasing *R* and looking for the qualitative change in behavior. Without trying to make a precise measurement, is the value of *R* that makes the circuit critically damped about what you expect from theory?

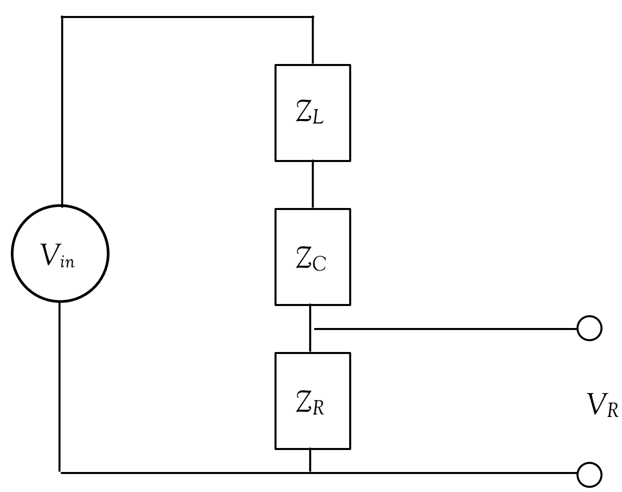
4. Driven Behavior- full measurement of the resonance curve for one case. Now drive the circuit with sine waves near the resonance frequency. Observe the behavior of the current as you sweep through the resonance frequency. Measure the resonance frequency and compare to that measured by plucking and to your calculated value (they should all agree!).

   Try increasing *L* in your circuit by a factor of 10, and make sure that the resonance frequency changes as you expect.

   Switch back to *L*= 1 mH (still with C= 1nF) and make *R*=100 Ω (chosen so that the resonance is not inconveniently narrow given the resolution of the frequency generator knob). Put the scope on x-y (as you did last week), set the channels to the same scale, and observe the behavior as you sweep the drive frequency through resonance.

   Now that you are warmed up, switch the scope back to the normal y-t mode, make a set of measurements of V_R vs frequency and plot. It is informative to look at your data on a graph where you have forced the frequency scale to start at zero – this gives you a sense of the width of your resonance curve compared to the center oscillation frequency (i.e. what's called the "*Q*" of the resonance). You don't have to print this, but do take a look.

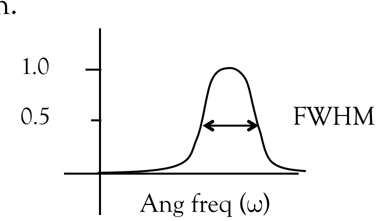
5. A quick theory break (this can be a collaborative effort!). For a lightly damped circuit there is a relatively simple formula for V_R^2 (proportional to the power dissipated in the resistor) as a function of frequency. Starting either from the diagram at right, or if you prefer, from eqn. 4.1.10 for the amplitude of a mechanical oscillator, find an approximate expression for $(V_R / V_0)^2$, valid when $Q \gg 1$. and $\omega_d = \omega_0 + \varepsilon$ with $\varepsilon \ll \omega_0$ (the idea being that for a lightly damped oscillator the response is essentially zero unless the drive frequency is very near the natural oscillation frequency). When you are done, you should have an expression of the form:



$$\left(\frac{V_R}{V_0}\right)^2 = \frac{1}{1+(\beta\varepsilon)^2}$$

   where β is a known constant. This function is called a Lorentzian.

   A convenient way to characterize the sharpness of a resonance curve is to measure the "Full width at half maximum" (FWHM)

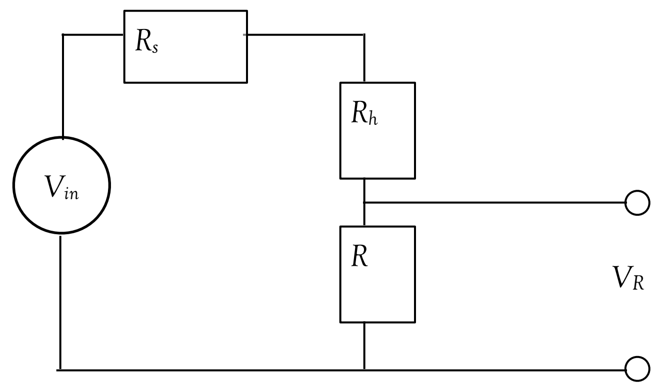


as shown in the sketch. Find the FWHM of the Lorentzian you just derived and show that your result is consistent with the general expression 4.3.5 (derived without making any approximations).

6. Replot the resonance data. In light of the calculation you just did, replot your existing data as V_R^2 vs frequency. What is the FWHM? What is measured Q of the oscillator ($Q = \frac{\omega_0}{FWHM}$) ?

   Now fit your data to a Lorentzian function. Find the find the effective resistance (which, I claim, will be a little bigger than expected).

7. More on the hidden resistance. You should have found that there is some "hidden" resistance in the circuit, i.e. a resistance higher than the sum of the output resistance of the signal generator, the resistor *R*, and the resistance of the coil. We would like to examine this quantity in more detail. One option would be to change parameters and then remeasure the resonance curve as you did above. While that would get us an answer, it would be a lot of work! Instead, let's note that on resonance the voltage across the capacitor and the inductor just cancel. At that frequency the circuit reduces to a simple resistive voltage divider as shown in the diagram. (Here I have explicitly shown the output resistance of the signal generator, R_S). With the scope you can measure the measure the voltage across R_h and *R* combined on one channel and across *R* alone on the other.



   It is convenient to make *R* small so that the effect of R_h is more pronounced, so let's make R=10 Ω. That done, measure the various voltages on resonance and find R_h (notice that R_S does not come into the equation). The value you get this way should be in reasonable agreement with that one you found from the width of the resonance curve.

   Replace the capacitor with a series of values - thus changing the resonant frequency of the circuit - and for each one measure R_h. (You might go from 1 nF to 100 nF, four or five points is probably enough). Plot up your data (R_h vs *f*) and see if it extrapolates to the DC value of resistance you measured for the coil.

   Since R_h depends on frequency it is clearly not simply an extra resistor hiding somewhere but is something more complicated. One effect that can lead to this kind of behavior is that with increasing frequency the current in a wire is not evenly distributed across the diameter of the wire but tends to be more on the outside. Thus the effective diameter of the wire decreases with frequency, leading to a resistance that increases with frequency. In addition, there may also be frequency dependent losses in the magnetic materials used in constructing the inductor. (A simple coil of wire would have to be physically large to get inductance values in this range so the inductor actually consists of wire wrapped around an iron core).

8. Resonant behavior of charge (displacement) . Thus far we have looked at the current flow in the circuit. Now rearrange the circuit to measure the voltage across the capacitor (keeping in mind that only one point in the circuit should be ground!). The voltage on the capacitor is proportional to the charge and hence analogous to the position of a mechanical oscillator. Choose a lightly damped case (say 1 mH, 1 nF, 10 Ω). Observe the phase behavior of V_C relative to the driving voltage. Draw phasor diagrams to explain the phase behavior for drive frequencies way below resonance, on resonance and way above resonance.

   Is it possible for V_C to be bigger than the drive voltage (try it!)? How is that possible, don't the voltages on the components have to add up to the drive voltage? On the problem set you will show that V_C on resonance should be the Q of the oscillator times the drive voltage. Now check your circuit and see if it is.

9. Stray capacitance. Put in a 100 pF capacitor and find the resonance frequency. From the measured frequency, calculate the capacitance. Try a 12 pF capacitor. Try just removing the capacitor entirely(!) Clearly there is some stray capacitance in the circuit.

   The black cable that you are using to connect the scope to the circuit is called a "BNC cable." The normal BNC cable has connectors (called, of course, "BNC connectors") like the one on the scope end of your cable. In the lab are some longer pieces of BNC cable and some barrel adapters that let you hook lengths of BNC cable together to make a longer cable. Get yourself a few meter long cable and an adapter and use this to lengthen the cable you are using to measure V_C. Now measure the resonant frequency of the circuit. What is the capacitance per meter of BNC cable?

10. Fourier Components – very quick preview. The *RLC* circuit acts a filter selecting only input signals near its special resonance frequency. As you may already know, a square wave can be made by adding up properly selected sine waves. If the signal generator is set to produce a square wave with a "frequency" of 1 kHz (say) it produces a square wave with the same period as a 1 kHz sine wave. Drive a lightly damped *RLC* circuit with a square wave with "frequency" near the circuit's resonance frequency. You should see that the circuit responds (i.e. has a large current flow) but oscillates as a sine wave not a square wave. Now lower the frequency of the square wave. I claim that when the square wave "frequency" is about 1/3 of the circuit's resonance frequency the circuit again shows a large response. We can describe this by saying that the square wave at "frequency" $f_0 / 3$ has a sine wave component at f_0 (i.e. at three times the fundamental frequency of the square wave). Can you see that the square wave also has a

component at 5 times its fundamental frequency, but not at 2 or 4 times?

Writeup (due next week at lab). You and your lab partner should prepare a joint report, written in collaboration. Your report should be a brief presentation of your results for sections 4,5,6,7 (the resonance curve + stray resistance). Assume that the reader has read (and understood) this lab sheet. The model I have in mind is not so much a formal report as a "lab notebook." You should write down enough information that a week from now you can come back and understand what you did and what conclusions you reached.