The Ontogeny of Quine's Ontology: Pythagoreanism, Nominalism, and the Role of Clarity

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1 Introduction

Quine's philosophical views did not emerge fully formed in the 1930s; rather, they changed over the seven decades he was philosophically active. This chapter investigates two episodes in Quine's ontological development: his engagement with Pythagoreanism (§2, with primary material in the Appendix), and his conversion from nominalism to Platonism about mathematics (§4). These two topics might seem completely distinct. However, although they could conceivably be treated separately, I will treat them together by considering the role *clarity* plays in both these episodes. Quine's changing views about the theoretical virtue of clarity, and which particular things are clear and which are not, help explain his ontological development. In particular, §4 offers a new hypothesis about the causes of Quine's conversion from nominalism to realism in which his views about clarity play an essential role. That said, Quine's later views about clarity, and its significance for theory choice, are complicated and arguably equivocal.

Information about Quine's Pythagorean investigations in the 1940s comes from a rich and fascinating notebook currently stored in the Quine archive, which Quine simply titled 'Logic Notes'. Before turning to Pythagoreanism and clarity, perhaps some information about the notebook is in order. It is 300 pages, and although the first entry is from May 1933, and the final one is dated January 1956, most entries date from the middle 30s. The majority of the notebook's pages cover material that is more logical or technical than philosophical, insofar as that distinction can be drawn. However, several entries take up more recognizably philosophical concerns, especially addressing the question of how certain philosophical issues connect to the technical work of both Quine and others. So one interesting feature of the 'Logic Notes' is that it reveals Quine's thoughts about the broader philosophical implications of his and others' logical work, even when those thoughts did not make it into print for whatever reason. The present chapter only covers a small part of the philosophical reflections found in Quine's notebook.

Paolo Mancosu has already done significant and insightful work on Quine's notebook. Mancosu (2008) draws on the 'Logic Notes' primarily to investigate the development of Quine's shifting ideas about nominalism, i.e. the denial of abstract entities. Quine first discusses the issue of nominalism in print in "Designation and Existence" (1939), and he endorses nominalism in a publication for the first and last time in 1947's "Steps Toward a Constructive Nominalism", jointly authored with Nelson Goodman (Quine and Goodman 1947). But as Mancosu's research using Quine's notebook makes clear, Quine was sympathetically exploring the tenability of nominalism as early as 1935, and these explorations continued behind the scenes for several years in various forms, leading up to publication in 1947.

Quine's notebook ranges over a wide variety of logical and philosophical topics. This chapter focuses on Quine's exploration of what he calls 'Neo-Pythagoreanism'. In 1940, Quine briefly considers writing a book developing this view. It should be stressed from the outset, however, that Quine does not wholeheartedly endorse Neo-Pythagoreanism in the notebook; rather, his goal is to craft a tenable version of it, and understand its advantages. The complete text of the relevant notebook entry is included as an appendix to this chapter.

2 Neo-Pythagoreanism: What and Why

What is Neo-Pythagoreanism, according to Quine? In a notebook entry entitled "Arithmetic and Nature", dated October 23-26, 1940, Quine begins with the following: "Under this title, a book expounding a Neo-Pythagoreanism wherein the positive integers constitute the whole ontology" (p. 224).[1] Since Quine had already articulated his dictum "[t]o be is to be the value of a variable" (Quine 1939, 708) by this time, for Quine this is equivalent to the position that all and only the natural numbers can be values of variables, i.e. the natural numbers exhaust the domain of discourse. Such a

[1]Throughout this chapter, page numbers without any further identifying information refer to page numbers in Quine's 'Logic Notes' notebook, W. V. Quine Papers, Houghton Library, Harvard University, MS Am 2587, Box 112: 3240.

view directly conflicts with nominalism (which, the notebooks show, Quine was exploring around this time), since nominalists hold that no abstract objects exist, whereas Neo-Pythagoreans hold that nothing exists except one kind of abstract object.

This Neo-Pythagoreanism is, to put it mildly, an unusual view. The Neo-Pythagorean does not countenance the existence of any mathematical objects other than numbers; so although every prime number exists, the *set* of all prime numbers does not. Even stranger, physical objects do not exist either. Or to put it another way (but no less strangely), the Neo-Pythagorean *can* claim that a particular set or physical object exists, but only if that set or physical object is identical to some natural number. For anything not identical to some natural number does not exist, on the Neo-Pythagorean view. Neo-Pythagoreanism will strike many as strange, to say the least.

But Quine's Neo-Pythagorean does not claim that arithmetic and its laws constitute the entirety of legitimate science, and that higher mathematics, physics, chemistry, etc. are akin to the study of the world of unicorns or Harry Potter. Rather, Quine wants reduce real analysis, physics, etc. to an arithmetical basis. That is, Quine wants to reconstruct our current successful formal and empirical sciences within the austere Neo-Pythagorean framework: set-theoretic concepts and physical concepts must be somehow defined within the language of arithmetic. This is analogous to Quine's conception of nominalism at the time: "[a]s a thesis in the philosophy of science, nominalism can be formulated thus: it is possible to set up a nominalistic language in which all of natural science can be expressed" (Quine 1939, 708). Neo-Pythagoreanism simply substitutes 'arithmetical' for 'nominalistic' in this formulation. Let us now turn to Quine's strategy for executing these two Neo-Pythagorean reconstructions.

How does a Neo-Pythagorean reduce mathematical entities to natural numbers? Quine invokes the (downward) Löwenheim-Skolem theorem, which states that any set of first-order sentences that has a true interpretation has a true interpretation whose domain is the natural numbers. So, in particular, there is an interpretation of the axioms of (first-order) set theory in which both all those axioms come out true, and the variables range over all and only the natural numbers. In this interpretation, '$\in$' cannot have its usual meaning (since on its standard meaning, '$n \in m$' will be false for all natural numbers n, m).

How does Quine's Neo-Pythagorean reduce physical objects to the natural numbers? His strategy is *not* to enumerate all of the physical objects in the universe, assigning each a natural number. Rather, Quine takes a page from Carnap's *Logical Syntax of Language*. There, Carnap distinguishes between traditional 'name languages' and modern 'co-ordinate languages,' and argues for the superiority of the latter for scientific purposes (Carnap 1934/1937, §3). The variables of a name language range over the more common-sense sort of spatiotemporal objects, such as individual people, animals, or even atoms. The individuals in a co-ordinate language, on the other hand, are standardly individual space-time points, represented by ordered quadruples of real numbers. The physical universe is described by attributing numerical properties to space-time points (e.g. the co-ordinate language sentence 'Temp(x_0, y_0, z_0, t_0, n)' expresses the same thing as the English expression 'The temperature at space-time point (x_0, y_0, z_0, t_0) is n)'),[2] and attributing numerical relations to pairs of space-time points (e.g. the proper time between two space-time points is m). The extension of 'x is water' will be the set of all space-time points where water is present. Of course, for a Neo-Pythagorean, that set must ultimately be replaced by a natural number. The next Neo-Pythagorean maneuver is probably obvious now: instead of thinking of the ordered quadruples of real numbers as *representing* physical space-time points, as we usually do, instead *identify* these quadruples with space-time points. And via an additional application of the Löwenheim-Skolem theorem, these n-tuples can in turn be re-interpreted by natural numbers.

This concludes, in very general outline, the basic ideas behind Quine's Neo-Pythagoreanism.[3] However, Quine does not rest content with the mere existence (guaranteed by the Löwenheim-Skolem theorem) of an interpretation of the set-theoretic axioms in the domain of the natural numbers that makes all of those axioms true. Rather, he attempts to sketch such an interpretation. First, he starts with a finite axiomatization of set theory,[4] and conjoins all the axioms into a single sentence, which he then converts into Skolem normal form. That is, each of the existential quantifiers in the giant conjunction is erased, and each existentially quantified variable x_i is replaced by a Skolem function f_i, which takes all the preceding universally

[2]More precisely, Quine notes that temperature applies only to regions of spacetime, not to individual points.

[3]Terminological note: Quine does not explicitly define 'Neo-Pythagoreanism.' Unless otherwise noted, I will use 'Pythagoreanism' without the prefix to refer to the view that all and only the natural numbers exist, and 'Neo-Pythagoreanism' to include, in addition, the particular strategy of using the Löwenheim-Skolem theorem to reduce all of mathematics and natural science to arithmetical concepts and claims. This is purely for my own expository convenience here; I make no claim that my stipulation accurately reflects how Quine himself would have used and understood those terms.

[4]The desire for finitude is presumably why Quine opts for a modified (p. 227) version of von Neumann-Bernays-Gödel (NGB) set theory over ZF or ZFC.

quantified variables as arguments. Then, Quine recursively specifies (in the classical metalanguage, not in the Neo-Pythagorean object language) a function g from positive integers to classes which makes essential use of these Skolem functions. (I realize this explanation is incomplete; for the full, gory details, see the Appendix, p. 225.) Because g maps several numbers to a single class, Quine cannot use g to *identify* classes and numbers, on pain of making distinct numbers identical. Thus, Quine takes only the smallest of all these numbers to be *the* class, in the Pythagorean language, to replace that class in standard set theory. Therefore, all the other natural numbers mapped to this same set will fail to satisfy 'x is a class', so that no class is identified with more than one natural number. Quine does not stop at constructing a mapping between natural numbers and classes. He also goes on to attempt an explicit surrogate notion of set membership (symbolized as '$\in'$' in the appendix below), such that for any two integers x, y: $x \in' y$ iff $g(x) \in g(y)$.

Quine foresees two possible problems with this project. First, although the axiom of choice guarantees that the Skolem functions f_i exist, we have no procedure for actually constructing them. (This concern is related to Quine's later, published reason for rejecting Pythagoreanism, viz. the proxy-function requirement on adequate reductions; more on this in §3.1 below.) Second, actually finding the arithmetical substitute $\in'$ for the usual notion of $\in$ may be difficult. However, Quine goes on to propose a rather complicated "first approximation" (p. 227) of $\in'$ (see p. 228 for details).

Instead of digging further into the technical details of Quine's Neo-Pythagorean project, let us switch our focus to his rationales for undertaking this project. There are at least two: clarity and ontological parsimony. Quine writes:

> this theory [Neo-Pythagoreanism] has—besides the virtues suggested on p. 224—the virtue of resolving the logical paradoxes without artificial or ad hoc postulation; for the postulates of the present logic will all be intuitively acceptable postulates about intuitively clear notions, viz. positive integers. (p. 230)

These two virtues are repeated in Quine's published discussion of Pythagoreanism: "there is an evident gain" in "reduc[ing] talk of sets ... to talk of natural numbers", "since the natural numbers are relatively clear and, as infinite sets go, economical" (Quine 1964, 211).[5] The 'virtues suggested

[5]"Ontological Relativity" also stresses the relative clarity of the natural numbers: "It will perhaps be felt that any set-theoretic explication of natural number is at best a case

on p. 224' are, I believe, ontological simplicity and its consequences. Although the quotation above does not explicitly articulate what 'the virtues suggested on p. 224' are, I think we can fairly safely infer that Quine has something like ontological parsimony in mind, for two reasons. First, as we just saw above, in his published discussion of Pythagoreanism, Quine identifies the two 'gains' Pythagoreanism offers as 'relative clarity' and 'economy.' Second, when we look at p. 224 itself, it simply states the broadest overview of the Neo-Pythagorean project. So "the positive integers constitute the whole ontology" must exhibit the theoretical virtue Quine has in mind here, and one virtue that this claim plausibly exhibits is (ontological) simplicity or economy: the Neo-Pythagorean does not have to accept the existence of sets or physical objects, so Neo-Pythagoreans have a more economical ontology than people who are also realists about sets or physical objects as well as natural numbers. Quine considered ontological simplicity a theoretical virtue for the remainder of his career—though later he realized that ontological simplicity often comes at the cost of *ideological* simplicity: ceteris paribus, the fewer entities a theory postulates, the more complex the concepts of the theory will be (Quine 1976c, 503). Several definitions on page 228 of the 'Logic Notes' provide clear examples of this phenomenon: in particular, the definition of '$\in$' is extremely complicated.

Let us now examine the virtue explicitly stated in the quotation above: the positive integers are especially intuitive and clear. Presumably, Quine thinks that arithmetical ontology and arithmetically-defined notions are clearer and more intuitive than that of set theories or various type theories proposed to 'resolve the logical paradoxes'. What does Quine mean by the terms 'intuitive' and 'clear'? Unfortunately, he does not explicate these notions further in his notebook. That said, he does appeal to them elsewhere, so perhaps these other uses will help us understand what he means by these terms in his 'Logic Notes'.[6] For example, in the pro-nominalism paper co-authored with Goodman seven years later, Quine writes: "Why do we refuse to admit the abstract objects that mathematics needs? Fun-

of *obscurum per obscurius* ... I must agree that a construction of sets and set theory from natural numbers and arithmetic would be far more desirable than the familiar opposite" (Quine 1968, 197). ('Obscure' is the standard contrast class for 'clear', and the next sentence in "Ontological Relativity" makes it clear that Quine has exactly that traditional opposition in mind.)

[6]See (Frost-Arnold 2013, §2.1.4) and (Ebbs 2016) for additional discussion of what 'clear' means for Quine. Ebbs suggests that 'clear' is roughly equivalent to 'common-sense' for Quine; I offer reasons to doubt that supposed equivalence in (Frost-Arnold 2016). But as I suggest below in §4.4, perhaps one can think of Quine's early criterion of 'clarity' as being transformed into later Quine's theoretical virtue of 'conservativeness'.

damentally this refusal is based on a philosophical intuition that cannot be justified by appeal to anything more ultimate" (Quine and Goodman 1947, 105). Here we see Quine thinking of intuitive claims as evidentially ultimate, not justified by anything further. And Quine expresses a similar point, in somewhat more detail, in a letter defending his sympathy for nominalism to Carnap, also written in 1947:

> I am not ready to say, though, that when we fix the basic features of our language... our guiding consideration is normally convenience exclusively. In my own predilection for an exclusively concrete ontology, there is something which does not reduce in any obvious way to considerations of mere convenience; viz. some vague but seemingly ultimate standard of intelligibility or clarity. (Creath 1990, 410)

This quotation delivers further points: first, clarity is at least roughly equivalent to intelligibility for Quine. And given that these two play the same role in this letter in justifying nominalism that 'philosophical intuition' plays in the published article, all three terms ('clear,' 'intelligible,' and 'intuitive') are perhaps roughly interchangeable for Quine. Second, just as in Quine's published article on nominalism, quoted above, clarity (etc.) is an 'ultimate standard': it is not reducible to 'convenience' or any other standard. And in a lecture from December 1940, Quine writes of reductive projects that "[i]n each case, if we do reduce, it is in order to reduce the obscure to the clearer" (Frost-Arnold 2013, 149). Of course, Neo-Pythagoreanism is a reductive project. This suggests that we can think of clarity as imposing a *direction* on (acceptable) explanations, for Quine: an explanation should explain the less clear in terms of the more clear (Quine 1968, 197).[7]

In the quotation from the notebook above, we saw Quine contrast 'intuitively clear' with 'artificial or ad hoc'. Why? We can better understand Quine's thinking here by turning to his 1947 pro-nominalism article co-authored with Goodman. There, Quine calls 'artificial' the various solutions invoked to avoid the logical paradoxes, paradoxes which arise from 'natural' or intuitive principles like Frege's basic axiom V. "Escape from these [logical] paradoxes can apparently only be effected by recourse to alternative rules whose artificiality and arbitrariness arouse suspicion that we are

[7] Thanks to Gary Ebbs for stressing the connection between clarity and explanation for Quine.

lost in a world of make-believe" (Quine and Goodman 1947, 105).[8] Quine says very similar things about the artificiality of type-theoretic restrictions used to avoid the paradoxes in (Quine 1947, 80-81) and (Quine 1941/1995, 27).[9]

In summary, in October 1940, Quine seriously entertained the idea of building a total language of science whose entire ontology comprised the natural numbers, because of both the greater intelligibility and ontological simplicity such a language offers. It is not clear from the notebook why Quine stopped actively pursuing the Pythagorean project shortly after he began it. However, 1940 was certainly not the last time Quine wrote about Pythagoreanism. The following section discusses Quine's later writings about Pythagorean projects.

3 Quine's later attitudes toward Pythagoreanism

Quine continues thinking and writing about Neo-Pythagoreanism after 1940. He completely rejects it in the 1960s, but reconsiders in the 1970s, when he reaches an ambivalent or mixed verdict about it. Examining Quine's later critical remarks will help us understand the 1940 Pythagorean project, and in particular, Quine's attitude toward certain obstacles for Pythagoreanism.

3.1 Rejection of Pythagoreanism in the 1960s

In "Ontological Reduction and the World of Numbers" (1964) and again (in slightly less detail) in "Ontological Relativity" (1968, 206-208), Quine explicitly describes a Neo-Pythagorean position. However, he rejects it unequivocally in both pieces: Quine claims that on the correct notion of theoretical reduction, "there ceases to be any evident way of arguing, from

[8]In the published "Steps toward a Constructive Nominalism", Quine and Goodman distinguish two rationales for nominalism: one is a 'philosophical intuition' which functions as an unjustified justifier, and the second (just seen) is that extant non-nominalist attempts to avoid the logical paradoxes are artificial. Quine's earlier remarks about Pythagoreanism and nominalism do not always explicitly distinguish between these two rationales. But they are distinct. 'Natural numbers (/concrete objects) are much easier to understand than sets (/abstract objects)' is a different consideration from 'To avoid the paradoxes, you must either be a nominalist or accept an artificial logic': we can imagine a theory whose domain of discourse is restricted to natural numbers or concreta making very artificial, ad hoc maneuvers to evade apparent disconfirmation.

[9]Sean Morris (2015) provides more details how Quine's view that set theory and type theory are artificial influenced his broader philosophical views.

the Löwenheim-Skolem theorem, that ontologies are generally reducible to natural numbers" (1964, 214). Comparing these published anti-Pythagorean pieces with Quine's logic notebook should prove mutually illuminating.

First and most obviously, in the two articles, Quine does not reveal or even suggest that he once entertained Pythagoreanism. Of course, this is not terribly surprising, and personally I am thankful that I can discuss a philosophical position in print without being required to confess whether or not I have ever held that position myself. That said, it is nonetheless noteworthy that Quine gives no hint that he himself toyed with the position. Instead, he chooses to associate it with George Berry (with qualifications): "Berry concluded that only common sense stands in the way of adopting an all-purpose Pythagorean ontology" (1964, 211). It is worth noting, however, that Berry's actual aim in the article Quine cites is to *refute* any sort of Löwenheim-Skolem-based argument for Pythagoreanism. Berry asks us to suppose that we have a true geographical theory.

> Would the fact that the original theory can be arithmetically interpreted lead anyone to claim that it *should* be, that the landmasses and peoples presupposed by the initial interpretation can justifiably be rejected and the whole earth considered, Pythagoras-wise, as a congeries of numbers? The existence of a given interpretation provides in itself no justification for preferring that interpretation to another. (Berry 1953, 53)

And he concludes from this that "the ontological significance of the Löwenheim-Skolem theorem would thus seem to be meagre indeed" (1953, 55)—a position very similar to Quine's in the 1960s. Furthermore, I do not think Berry's point is best described as 'Only common sense prevents us from accepting Löwenheim-Skolem-driven Pythagoreanism.' Instead, Berry's point is that all the Löwenheim-Skolem theorem delivers is that a particular numerical interpretation exists for any set of first-order sentences, but the fact that a set of sentences *can* be interpreted one way does not entail that it *should* be interpreted that way. Berry's point may be as simple as: Mount Kilimanjaro is not a number (since it has spatial dimensions, and an average temperature, etc., traits which numbers lack). Perhaps that is mere 'common sense', but if so, then it is not obvious which truths would *not* count as common sense. So not only does Quine give no hint in the 1960s that he was once sympathetically investigated 'an all-purpose Pythagorean ontology', but he associates that view with someone who rejects it at least as strongly as Quine does.

Second, let us examine Quine's published rejection of Neo-Pythagoreanism in the 1960s. The fundamental reason for this rejection is that, according to Quine, a successful reduction of one theory T_1 to a second theory T_2 requires what Quine calls a 'proxy function,' and "the Löwenheim-Skolem Theorem determines, in the general case, no proxy function" (Quine 1964, 216). A proxy function is "a function whose values exhaust the old things [the range of values of the variables of T_1] ... as their arguments range over the new things," the range of values of the variables of T_2 (1964, 214). Why does imposing a proxy-function requirement create a problem for Neo-Pythagoreanism? If we are trying to reduce set theory to arithmetic, then we will need a total function from sets into natural numbers. This makes the problem for Pythagoreanism obvious: by Cantor's theorem, there are more sets than natural numbers, so any (total) function from the sets to the natural numbers will have to assign the same number to distinct sets. But then the supposedly reducing theory does not adequately capture or match the old one: the arithmetized theory says that two things are identical which the original set theory says are distinct.

Why did Quine not consider this issue of mismatched cardinalities between sets and natural numbers a problem for Pythagoreanism in October 1940? The easy answer is that he did not hold his proxy-function requirement yet, and the Löwenheim-Skolem theorem guarantees that there is a numerical interpretation that makes the axioms of (first-order) set theory true. Thus, a nondenumerable universe is not necessary to make the axioms of set theory true. Thus, Pythagoreans do not need a *total* function from the sets to the natural numbers (that is, the function g described above, from the natural numbers to the sets, need not be onto). The 'extra' sets can be ignored, without losing any of the theorems of set theory. Thus, the fact that g is not onto is not a problem for Pythagoreans, unless they also adopt the proxy-function requirement on reductions.

Interestingly, Quine has another rationale at his disposal in 1940 to soothe cardinality worries about reducing sets to natural numbers, but he does not use it in his discussions of Pythagoreanism. As mentioned above, Quine was exploring the viability of nominalism during the late 1930s in his logic notebook. This nominalism had a more medieval flavor than the eventual nominalism that was published in the co-authored article with Goodman, as Mancosu (2008) noted. Quine's early version of nominalism sought to replace universals and classes with their names, i.e. their *nomina*: "we are perhaps justified in reopening the question of the nominalistic identifiability of classes with terms (expressions). From the classical point of view, this course is blocked by the fact that expressions can be correlated with the natural numbers (lexicographically) whereas classes cannot" (p. 209). The reason Quine thinks it may now be 'justified' to attempt to identify terms with classes is that in his own logical system of New Foundations (NF), Cantor's theorem does not hold.[10] (Reminder: Cantor's theorem entails that the set of natural numbers is strictly smaller than the set of all sets of natural numbers.) So Quine's idea is that "perhaps the alleged class which would be cited as violating any given correlation of terms and classes," i.e. the 'diagonal set' at the heart of the proof of Cantor's theorem, "is actually a spurious class" in NF, because it could not be expressed by any well-formed formula of NF. Now, this quotation is of course about nominalism, not Pythagoreanism, but the point carries over directly, substituting 'natural numbers' for 'terms': Quine might have dispelled some of the worries about the (supposedly) mismatched cardinalities of sets and natural numbers, if he had couched his Pythagoreanism within the framework of NF instead of NGB set theory. But Quine does not mention this in the section of his notebook on Pythagoreanism. Perhaps Quine omits this because he knows a theorem very similar to Cantor's Theorem does hold in NF (see footnote 10), but that is speculative.

Finally, before leaving the Quine of the 1960s, it is worth noting that although Quine rejects Neo-Pythagoreanism in 1964 on the grounds that it fails to reduce set theory to arithmetic, the other half of the Neo-Pythagorean project, which reduces physical entities to mathematical entities, emerges unscathed. That is, in the 1960s, he still endorses as legitimate "the reduction of an ontology of place-times to an ontology of number quadruples" (1964, 215), i.e. the reduction of natural science to mathematics described in 1940 in the his notebook. So although in the 1960s Quine believes Neo-Pythagoreanism is not a global success, it enjoys a local or partial success nonetheless. This leads naturally to the last topic of this section, namely, Quine's views about Pythagoreanism in the 1970s and beyond.

3.2 Quine's Hyper-Pythagoreanism in the 1970s

Quine's strict rejection of 'blanket Pythagoreanism' in 1964 softens somewhat in the 1970s, in particular in "Whither Physical Objects?" (1976c). Quine calls the view he develops there "hyper-Pythagoreanism",[11] but this

[10] However, a theorem very similar to Cantor's theorem does; this is why Quine says that Cantor's theorem has an "ambiguous position" in NF (p. 209).

[11] See Kemp (2017) for much more on Quine's hyper-Pythagoreanism, and an argument that "Quine could and perhaps should have accepted hyper-Pythagoreanism, without relinquishing any of his main commitments" (155).

is not the view that the ontologies of all natural and formal sciences can be reduced to natural numbers. Rather, it is that the ontology of each of these theories can be reduced to *sets*. Because hyper-Pythagoreanism uses sets instead of natural numbers, his argument for it does not mention the Löwenheim-Skolem theorem. Rather, the argument is based upon considerations from physical theory. As Quine puts it: "Our physical objects have evaporated into mere sets of numerical coordinates. This was an outcome ... of physics itself". Similarly: "we ... find ourselves constrained to this anti-physical sort of reductionism from the side of physics itself" (1976c, 502). And then the numbers in the 'numerical coordinates' can be reduced to sets, in the familiar way.

What is this physics-based argument for set-theoretic Pythagoreanism? Quine begins with the initial starting-point of any physical investigation, namely, matter. But because of various sorts of quantum-mechanical strangeness (Quine 1979, 164), "the notion of matter wavers when we get down to the level of electrons" and other fundamental particles (1976c, 498). So "[m]atter goes by the board," and is replaced by "a distribution of states over space-time" (1976c, 499), echoed in (Quine 1979, 164). (The fact that material bodies in a quantum-mechanical world have counter-intuitive properties plays an important role in the next section of this chapter, in connection with the theoretical virtue of clarity and Quine's rejection of nominalism.) But to describe these states (e.g. the temperature of a region), we irreducibly need numbers, so nominalism fails. But Quine then jumps immediately to set theory as "a familiar way of integrating the whole universe of mathematics" (1976c, 500); he does not consider here the properly Pythagorean possibility that we can make do with natural numbers but not sets. And "once we have reluctantly admitted all the ontology of set theory, ... we can thereupon dispense with the other part of our ontology, the space-time regions," as well as all the other mathematical objects like numbers (1976c, 500-501), for all the same reasons discussed earlier in this chapter: we simply identify each space-time point with a quadruple of real numbers.

I do not want to analyze Quine's physics-driven rationale for hyper-Pythagoreanism. Rather, I wish to close this section by highlighting Quine's ambivalence towards this form of Pythagoreanism. He describes the admission of sets as "melancholy," something only done "reluctantly" (1976c, 500). This fits with his post-nominalist position that "Nominalism... would be my actual position if I could make a go of it" (1986, 26). His ambivalence is clearest when he writes: "[o]ur ontological débacle, if débacle it be, is a triumph of hyper-Pythagoreanism" (1976c, 503), cf.

(Quine 1979, 164). Here, Quine is non-committal about whether hyper-Pythagoreanism is a disaster. Hyper-Pythagoreanism makes another appearance a few years later, in "Things and their Place in Theories," but Quine does not endorse it there. Rather, he uses it as one of four examples of reductive projects, in service of his overall thesis of the inscrutability of reference (Quine 1981). In short, both early and late, Quine never wholeheartedly endorses Pythagoreanism—either the neo-Pythagoreanism of 1940 or the hyper-Pythagoreanism of the 1970s—but he often finds it somewhat appealing as well.

4 Quine's Flight from Nominalism

I now turn to what might at first appear to be a completely unrelated topic: Quine's conversion from nominalist to Platonist, i.e. a realist about set theory. However, I hope to show that we can better understand Quine's attitudes towards both Pythagoreanism and his shift from nominalism to Platonism by paying closer attention to Quine's views about clarity (or intelligibility, or naturalness), and how they change over time. Specifically, I will advance a new hypothesis about the causes of Quine's conversion from nominalism, in which clarity plays a central role. But before we discuss his repudiation of nominalism, we should first outline his views concerning nominalism leading up to his conversion.

4.1 Quine's Nominalism

As noted above (§1), Quine explored nominalism from the mid-1930s onward in his private notebook, but didn't endorse nominalism in print until 1947's "Steps Toward a Constructive Nominalism," co-authored with Goodman. Mancosu argues that the beginning of Quine's "hopeful engagement" with nominalism was a 1939 lecture at the Fifth International Congress for the Unity of Science, held in Cambridge, Massachusetts (Mancosu 2008, 32). And in "Designation and Existence," published that same year, Quine articulates one version of the nominalist's central thesis: "it is possible to set up a nominalistic language in which all of natural science can be expressed" (1939, 708). (A language counts as nominalistic only if it ineliminably quantifies only over concrete things.) And similarly sympathetic yet non-committal remarks appear in 1944's *O Sentido da Nova Logica* (Janssen-Lauret 2018). But neither of these earlier publications, nor the 1939 lecture, endorses nominalism wholeheartedly: Quine leaves it an open question until 1947's article with Goodman.

Quine's investigations into nominalism were boosted in the Spring semester of 1941. During that academic school year, he, Alfred Tarski, Rudolf Carnap, and Nelson Goodman were all at Harvard together. They met frequently to discuss several topics of shared interest, and Carnap took dictation notes in secretarial shorthand.[12] The topic they discussed most often was how to construct a finitist and nominalist language adequate to the purposes of science. Quine even tells us in his published defense of nominalism, seven years later, that the original impetus for his form of nominalism was this set of conversations (Quine and Goodman 1947, 112 [fn. 12]). This project was proposed by Tarski. He says: "at bottom, I only understand a language that meets the following conditions": it is first-order, and the domain of quantification includes all and only the 'physical things'. And since there might be only a finite number of physical things in our universe, we cannot assume the domain is infinite (Frost-Arnold 2013, 153). (The particular conditions underwent slight modifications over the course of the semester of discussions; this is their final form.) These criteria are consistently presented as conditions on what is understandable or intelligible [*verständlich*]. The obvious next question to ask is: what does Tarski (and the other discussants) mean by 'intelligible'? Unfortunately, the discussants do not address this question in detail in Carnap's dictation notes. Some attempts at answering it can be found in (Frost-Arnold 2013, §2.1).

As discussed above (at the end of §2), one can reasonably say is that Quine thinks of 'intelligible' as roughly equivalent to 'clear.' We saw earlier that in the 1930s and 40s, Quine does consider clarity as a fundamental theoretical virtue, distinct from mere convenience: "in my own predilection for an exclusively concrete ontology, there is. . . some vague but seemingly ultimate standard of intelligibility or clarity" (Creath 1990, 410). We also saw that it seems to be closely tied to intuitiveness for him. Furthermore, this collection of terms ('intuitive,' 'clear,' 'intelligible') also appears to be closely associated with "epistemological immediacy" for Quine in the mid-1940s (Frost-Arnold 2013, 35). And he explicitly ties this virtue of clarity to the nominalist project he and Tarski were investigating in 1941: in a 1943 letter to Carnap, Quine writes that he and Tarski "urged that the only logic to which we could attach any seeming epistemological immediacy would be some sort of finitistic logic" (Creath 1990, 295) ('finitistic logic' was one name the discussants used for a language meeting Tarski's intelligibility

[12] These notes were analyzed first in (Mancosu 2005). Frost-Arnold (2013) includes a complete German transcription of Carnap's shorthand notes, and an English translation.

requirements). In short: one of the virtues of a nominalist language is that it is intelligible, clear, and/or its logic is epistemologically immediate.

4.2 Quine's Break with Nominalism: the Standard View and its Shortcomings

Now let us skip forward a few years, to Quine's renunciation of nominalism. This happened publicly at the end of in 1948, in the closing paragraphs of "On What There Is." Paolo Mancosu has argued compellingly that we can point to an earlier, more specific moment as Quine's break from nominalism, specifically, a March 1948 letter from Quine to J. H. Woodger, published in (Mancosu 2008).

Let us suppose Mancosu is correct about the date of Quine's repudiation of nominalism. The next question to ask is: *why* did Quine abandon nominalism at this point in time? Quine, in later years, says that he rejects nominalism because it does not capture enough classical mathematics to do natural science, and therefore Quine is unwilling to part with so much classical mathematics.[13] So one natural conclusion to infer from this is that (a) Quine finally realized nominalist languages were seriously impoverished around March 1948, (b) this realization caused him to renounce nominalism and instead accept full classical mathematics under a realist or Platonist interpretation, (c) because he thought full classical mathematics understood realistically is necessary to pursue natural science.[14] I will call (a)-(c) the 'Standard Hypothesis' for why Quine gave up nominalism for realism in March 1948.

I think the Standard Hypothesis has two problems, and as a result is at best insufficient to explain Quine's switch, and at worst false. First, Quine was already aware that nominalist scruples would impose serious limitations upon classical mathematics, and therefore also on the natural sciences

[13]When explaining why he is not a nominalist in *Word and Object*, Quine points to the expressive power of full classical mathematics. He writes that the notion of a class "confers a power" of expression "that is not known to be available through less objectionable channels" (1960, 267), and that "[t]he reason for admitting numbers as objects is precisely their efficacy in organizing and expediting the sciences" (1960, 237). For example, Quine says that "sets can pay their way ... by the definition of the *closed iterate* of a two-place predicate. ... For every predicate in our language we can express also its closed iterate, if we allow ourselves to quantify over sets as values of our variables" (1979, 161).

[14]John Burgess explicitly articulates part (c): "it does really seem to be Quine's view that only the *indispensable necessity* of positing mathematical entities in science can justify belief in them; the mere fact that such posits are a *customary convenience* is not enough for Quine" (Burgess 2014, 291).

using that mathematics, throughout his engagement with nominalism from the mid-1930s onward. For example, in 1939, Quine's notes for his lecture to the Congress of the Unity of Science summarize his position as follows:

> Strong argument for nominalism. Probably can't get classical mathematics. But enough mathematics for physical science? If this could be established, good reason to consider the problem solved. (Mancosu 2008, 32)

So even in 1939 Quine strongly suspects that a nominalist cannot account for all of classical mathematics. And in the eventual published version of this lecture, Quine also says it is "likely" that "fragments of classical mathematics must be sacrificed under all such [nominalist] constructions"; and he again proposes that one could still be a nominalist, by showing that the sacrificed portions are "inessential to science" (1939/1976a, 202). Precisely the same idea recurs in the 1941 conversations with Tarski (Frost-Arnold 2013, 177). And Quine's published defense of nominalism show Quine realized how very severe the nominalist restrictions are:

> For example, the formula which is the full expansion in our [nominalist] object language of '$(n)(n + n = 2n)$' will contain variables calling for abstract entities as values; and if it cannot be translated into nominalistic language, it will in one sense be meaningless for us. But, taking that formula as a string of marks, we can determine whether it is indeed a proper formula of our object language, and what consequence-relationships it has to other formulas. We can thus handle much of classical logic and mathematics without in any further sense understanding, or granting the truth of, the formulas we are dealing with. (Quine and Goodman 1947, 122)

So Quine, in his fully nominalist phase, is even willing to accept that '$(n)(n + n = 2n)$' is meaningless (and thus untrue). He also recognizes that certain intuitively valid proofs will not be valid in a nominalistically acceptable logic, because we will run out of 'ink' before the proof can be completed; for example, if all the material in universe is used up by writing out the two premises ϕ and ψ, then $\phi \wedge \psi$ is unprovable from those premises (121). This makes part (a) of the Standard Hypothesis unappealing: Quine already believed (perhaps with varying degrees of conviction) from 1939 to 1947 that mathematics that honors nominalist scruples could not capture the truth of full classical mathematics, or even merely full classical logic. This also undermines part (b) of the Standard Hypothesis: if the reason Quine became a Platonist about full classical mathematics is that he realized nominalist mathematics would be seriously impoverished, then Quine should have become a Platonist in 1939 (or at least before 1947). For this reason, I think the Standard Hypothesis is insufficient at best.

A defender of the Standard Hypothesis could reply to this charge by using the quotations in the immediately preceding paragraph to clarify or refine the Standard Hypothesis. Quine renounced nominalism in 1948, because he finally came to realize that the nominalist cannot capture "enough mathematics for physical science," or put otherwise, that the parts of mathematics the nominalist must forsake are *not* "inessential to science". In other words, Quine finally lost hope that nominalism could be formulated in a way that could capture enough of classical mathematics to support natural science. However, I don't think this reply will work. The first reason is the instrumentalist attitude Quine takes towards mathematics in the above quote from the 1947 paper. He and Goodman go on to say that the nominalist can think of classical mathematics as an abacus, and no one considers an abacus to be true. So in his full-blown nominalist phase, he was content with very little of classical mathematics being true. Second, I cannot find, in Quine's public or private writings of the late 1940s or early 50s, any explicit argument that some particular piece of classical mathematics M is essential to science, yet nominalism cannot appropriately capture M, or Quine explicitly arguing that someone who takes an instrumentalist, i.e. abacus-like, attitude towards classical mathematics cannot perform a particular important bit of science. But if this reply on behalf of the Standard Hypothesis were correct, presumably we would see some evidence that Quine reasoned like this. This absence of evidence is of course not conclusive disproof. Nonetheless, it is partial evidence against this refined version of the Standard Hypothesis for why Quine renounced nominalism in early 1948.

There is a second plausible reason why the Standard Hypothesis for Quine's switch from nominalism to Platonism may be inadequate. Large reconstructive or reductive projects, such as the project to capture as much of classical mathematics as possible within a nominalist language, take a massive amount of time, ingenuity, and (often) collaborators. For example, think of the enormous amounts of energy invested in the logicist project to reduce mathematics to logic in the early part of the 20^{th} century: many great minds devoted many years to the endeavor. And this work required creativity and originality for breakthroughs, such as Frege's definition of 'number'. Of course serious conceptual and practical obstacles arose: the discovery of Russell's paradox, and the enormous amount of time sunk into *Principia Mathematica*, among others. But Frege, Russell, Whitehead, Hilbert and others did not give up on the project, even when setbacks like Russell's paradox arose. Compare this to Quine's nominalist project. A few months after publishing the first article with Goodman in 1947, he repudiated it. It is of course possible that at the end of 1947 or beginning of 1948 Quine came to believe that no amount of intellectual energy, ingenuity, or technical brilliance would generate a nominalistic reconstruction of enough classical mathematics (more specifically, a nominalist reconstruction of the realistic proof theory the 1947 Goodman-Quine version of nominalism needs[15]). But that surrender seems too quick, especially if we compare this nominalist project to the logicist one, which was of course very familiar to Quine. Additionally, there does not appear to be anything in the nominalist case analogous to Gödel's incompleteness theorems, which sounded the death-knell for (classical) logicism.[16] I do not wish to place too much weight on this consideration; it does not conclusively disprove the Standard Hypothesis for Quine's renunciation of nominalism, but it does cast some additional doubt on it.

4.3 An Alternative Explanation of Quine's Break with Nominalism: the Clarity Hypothesis

If the Standard Hypothesis is not the best explanation for Quine's shift, what is? I will argue for an alternative position, which I call the 'Clarity Hypothesis.' This position has two planks: first, Quine gives up nominalism in March 1948 because he stopped thinking nominalism enjoys what he previously considered its primary theoretical virtue, namely, clarity. Second, Quine stopped thinking that because modern science suggests that concrete objects, the nominalist's paradigm examples of clarity, are actually unclear. A further addendum, which is not part of the Clarity Hypothesis per se, is that post-nominalist Quine often seems to downgrade

[15]Quine and Goodman's 1947 paper adopts a global instrumentalism about any mathematics that quantifies ineliminably over abstract objects, but their instrumentalism needs to interpret proof theory—the instrument—realistically: "we can understand" the terms and rules of the proof theory "in the strict sense that we can express them in purely nominalistic language" (1947, 122).

[16]There are of course many counter-intuitive features of Quine and Goodman's nominalism; e.g., as mentioned above, the rule of $\wedge$-introduction is not universally valid. But the co-authored article with Goodman shows that Quine was already aware of many of the worst of these counter-intuitive consequences in 1947. My point here is that there is no *new* logico-mathematical discovery around this time that could have reasonably tipped the balance away from nominalism and towards Platonism.

the importance of clarity as a theoretical virtue in general. That is, whereas before 1948 Quine would place greater weight on the clarity (intelligibility, intuitiveness, naturalness) of a theory when attempting to choose between theories, afterwards Quine places much less (and perhaps no) weight on a theory's clarity, focusing instead on its other virtues, such as its empirical adequacy and simplicity. I will call this the 'Addendum' to the Clarity Hypothesis. I am separating it, because the case for it is both more vexed and more complex than the case for the Clarity Hypothesis (the difficulties and complexities are outlined in §4.4).

The primary evidence for the Clarity Hypothesis comes from the aforementioned March 1948 letter from Quine to Woodger. I will quote from it extensively.

> [T]he question what ontology to accept is in principle similar to the question what system of physics or biology to accept: it turns finally on the relative elegance and simplicity with which the theory serves to group and correlate our sense data. We accept a theory of physical objects... because this gives us the neatest and most convenient filing cabinet yet known in which to file away our experiences. Now the positing of abstract entities (as values of variables) is the same kind of thing. As an adjunct to natural science, classical mathematics is probably unnecessary. (Mancosu 2008, 43)

First, note that this is evidence against plank (c) of the Standard Hypothesis. This plank asserts that Quine became a Platonist about classical mathematics because he thought classical mathematics was necessary for the absolutely central scientific practices of theorizing and prediction, and a nominalist could not accept enough classical mathematics to engage in these practices sensibly. The above quotation indicates that, at least at the moment of Quine's conversion, this was not his rationale. For even here, at the moment he renounces nominalism, Quine says classical mathematics is 'probably' not necessary to achieve the goals of natural science; it is still 'inessential'. This suggests that the Standard Hypothesis is not merely an incomplete explanation of Quine's shift from nominalist to Platonist, but straightforwardly incorrect. Rather, in this letter, Quine is inclined towards Platonism because it leads to a "simpler and more convenient" mathematics than nominalism.

Furthermore, note that in the above quotation, the supposed clarity of nominalism does not appear as a criterion in the decision of which ontology to adopt. Recall that according to Tarski, the point of engaging in the nominalist project is that only a nominalist language is truly intelligible. And similarly we saw Quine say in a 1947 letter that his "predilection for an exclusively concrete ontology" was based on something that does "not reduce ... to considerations of mere convenience," but instead relies on a "standard of intelligibility or clarity." In the above quotation, however, the convenience of classical mathematics over attempted nominalistic substitutes decides the question in favor of Platonism.

Quine continues:

> ... classical mathematics is probably unnecessary; still it is simpler and more convenient than any fragmentary substitute that could be given meaning in nominalistic terms. Hence the motive—and a good one—for positing abstract entities which classical mathematics needs.... The platonistic acceptance of classes leads to Russell's paradox et al., and so has to be modified with artificial restrictions. But so does the acceptance of a physical ontology, in latter days, lead to strange results: the wave-corpuscle paradox and the indeterminacy. (Mancosu 2008, 43)

The key point to extract from the this quotation concerns the naturalness or clarity—or rather, the lack thereof—of nominalism. A physical body, the paradigmatic 'concrete object' of nominalism, becomes "strange" in modern physics. So in this letter, Quine says that the concept of a physical body simply is not as clear as he (and Tarski) had once believed. Quine spells out a similar line of thought in "Whither Physical Objects?", discussed above. There, he argues that the strange and counterintuitive features of quantum particles (e.g. the apparent non-individuality of quantum objects) renders the concept of a physical particle "tenuous"; thus "[m]atter evidently goes by the board", to be replaced by fields (Quine 1976c, 498-499); very similar phrasing appears in (Quine 1979, 164).[17] This is directly relevant to Quine's conversion to Platonism: if the supposed primary attraction of nominalism is the naturalness or clarity of its ontology, but the modern physicists inform us that the values of nominalist variables—which include physical particles—are unclear, then the main reason for being a nominalist disappears. The nominalist is left with a mathematical system that is in many

[17]Quine's appeal to quantum strangeness leads to somewhat different ultimate conclusions in the 1948 letter to Woodger on the one hand, and "Whither Physical Objects?" and "Facts of the Matter" on the other. In the former, the concept of a physical object is unclear, whereas in the latter, material objects are dropped from the domain of discourse in favor of fields and their attributes (which are ultimately jettisoned in favor of sets).

ways more complicated than the classical mathematician's, but now without something positive to offset those disadvantages. Furthermore, recall that Quine and Goodman consider the 'unnatural,' 'artificial,' and 'arbitrary' aspects of type theories and set theories designed to avoid the paradoxes to be evidence against interpreting those theories realistically (1947, 105). But if the physicists are willing to accept a theory as strange and unnatural as quantum mechanics, then perhaps the logicians and philosophers should be willing to accept the supposedly unnatural or artificial type theories and set theories that can capture full classical mathematics. This point will be explored further in §4.4, in the discussion of whether and to what extent post-nominalist Quine still considers clarity of concepts to be a theoretical virtue.

Now, at this point, someone who had read §2 above might wonder: 'If Quine thinks quantum mechanics casts doubt on the clarity or naturalness of the concept of a physical object in 1948, then instead of endorsing full set theory, why doesn't he instead fall back on just the natural numbers, in line with his Neo-Pythagoreanism, which he also pursued because the natural numbers are paradigmatically clear?' I do not have an airtight answer to this; presumably whatever reasons led him to stop working on Neo-Pythagoreanism in late 1940 will be part of the answer. But at least one other text is relevant here. In "Ontological Relativity," Quine says that Gödel's first incompleteness theorem, and non-standard models of the arithmetic, suggest that even the natural numbers are not entirely clear:

> [o]ur impression of the clarity even of the notion of natural number itself has suffered somewhat from Gödel's proof of the impossibility of a complete proof procedure for elementary number theory, or ... from Skolem's and Henkin's observations that all laws of natural numbers admit nonstandard models. (1968, 197)

Of course, "Ontological Relativity" appears two decades after the 1948 letter to Woodger, so we cannot infer that Quine had exactly these motivations in mind when he moved all the way to set-theoretic Platonism instead of stopping at arithmetical Platonism, i.e. Pythagoreanism. However, of course, the relevant results of Gödel and Skolem, mentioned in the quotation above, were certainly available to Quine in 1948, so it is at least possible that Quine had something like this later reasoning in mind.

If the Clarity Hypothesis is correct, then a further historical question naturally presents itself: given that quantum mechanics had been on the intellectual scene for decades, why didn't Quine have this idea about the unclarity of material bodies before 1948?[18] (Carnap had already pointed out that quantum physicists had given up on achieving "intuitive understanding" of material bodies almost a decade earlier (1939, §25).) I do not have a smoking gun; often the human mind simply takes a long time to connect two ideas that it has stored for years. But Quine's autobiography suggests a partial proximate cause. On March 15, 1948 (just a few days before Quine sends the letter to Woodger), Quine gave a talk at Princeton. At lunch that day, he spoke with Niels Bohr: "[h]e was explaining his complementarity, which was unfamiliar to me" (1985, 200). So perhaps this conversation with Bohr made quantum-mechanical considerations more salient to Quine in March 1948.

Thus far, I have argued that the Clarity Hypothesis makes better historical sense of Quine's shift from nominalist to Platonist than the Standard Hypothesis. But might there be other historical explanations of Quine's conversion that are even better than the Clarity Hypothesis? Sander Verhaegh recently presented an alternative explanation (2016, §7). According to Verhaegh, Quine gives up nominalism for realism in the late 1940s because Quine changed from being a 'narrow-scoped holist' to a 'wide-scoped holist' at this time. In Verhaegh's terminology, narrow-scoped holism is the view that only the *natural* sciences face the tribunal of observation as a whole; whereas wide-scoped holism holds that *all* the sciences, including the formal sciences of logic and mathematics, face the tribunal of observation as a whole. He writes:

> Quine's ... acceptance of a wide-scoped holism provided him with an argument for allowing abstract objects. If we evaluate our logical and mathematical theories in terms of their contribution to our best scientific theories, dismissing any extra-scientific justification in terms of analyticity, then there is no reason not to treat physical and mathematical objects on a par. ... While the early Quine was a realist about physical objects but did not yet want to fully commit himself to the abstract objects of mathematics for philosophical reasons, his acceptance of wide-scoped holism in the late 1940s seems to have removed his reasons not to extend his realism to abstract entities. (Verhaegh 2017, 336)

This quotation contains several interesting claims, but I will focus on one in particular: wide-scoped holism gives Quine (and most other people) rea-

[18] Thanks to Greg Lavers for explicitly raising this question.

son to reject nominalism and accept Platonism. I think this is contestable.

I agree that Quine became a wide-scoped holist; I deny that this is the best explanation for his shift from nominalism to Platonism. First, as Verhaegh himself suggests (§5), Quine may have received his initial impetus for wide-scoped holism from Tarski in the 1941 conversations at Harvard (Frost-Arnold 2013, 87). However, Tarski was also the person who introduced and most energetically pushed nominalism in those conversations—so the historical causal connection between wide-scoped holism and rejecting nominalism is obscure. Second, there is not a strong conceptual tie between Platonism and wide-scoped holism. One could be a wide-scoped holist and a nominalist (presumably, this was Tarski's position): imagine a circumstance in which a nominalist reconstruction of mathematics, such as that described in (Field 1980), worked to everyone's satisfaction, and there was little to no scientific benefit to assuming that non-concrete entities exist in addition to the concrete ones. Such a person would be a wide-scoped holist who was also a nominalist. For these two reasons, Quine's shift to wide-scoped holism does not seem to me to be the best explanation for his rejection of nominalism.

There is another hypothesis, besides the three just considered (viz. the Standard, Clarity, and Wide-scope hypotheses), for the timing of Quine's renunciation of nominalism: his turn to naturalism. In a 1946 lecture on nominalism, Quine says:

> classical mathematics is part of science; and I have said that universals have to be admitted as values of its variables; so it follows that the thesis of nominalism is false. What has the nominalist to say to this? He need not give up yet; not if he loves his nominalism more than his mathematics. He can make his adjustment by repudiating as philosophically unsound those parts of science which resist his tenets. (2008, 17)

One way to be a nominalist, this quotation shows, is to declare the parts of classical mathematics that quantify ineliminably over abstract objects 'philosophically unsound'. But allowing philosophical considerations to trump scientific ones, i.e. 'loving nominalism more than mathematics', is what Quine will later call 'first philosophy'—it is anti-naturalistic (1981, 67, 72). So this fourth hypothesis, call it the 'Naturalism Hypothesis', holds that Quine rejected nominalism because he became a naturalist, and nominalism is incompatible with naturalism, since favoring nominalism would require a special philosophical criterion, distinct from those of the empirical and formal sciences, which can outweigh scientific considerations.

This Naturalism Hypothesis cannot be completely correct, because Quine's rejection of nominalism in 1948 apparently pre-dates his embrace of naturalism. As Verhaegh has argued, Quine did not reach his full, mature naturalism in a version that would justify rejecting nominalism until 1952 (Verhaegh *forthcoming*, §8). That said, *after* explicitly accepting naturalism, Quine could thereafter appeal to his naturalism to formulate an argument against his earlier nominalism: recall from above that Quine describes his and Goodman's ultimate motivation for nominalism as 'based on a *philosophical* intuition that cannot be justified by appeal to anything more ultimate'. If that motivation really is properly philosophical, i.e. extra-scientific, and the mature Quinean naturalist recognizes no extra-scientific theoretical virtues, then Quine's fundamental justification for nominalism evaporates. So it could well be that the argument from quantum strangeness against nominalism becomes otiose after 1952, when Quine explicitly adopts naturalism.

That said, there might be an alternative formulation of the Naturalism Hypothesis that is consistent with the Clarity Hypothesis. On this alternative formulation, Quine's rejection of nominalism in the March 1948 letter to Woodger is the moment he becomes a naturalist—even if Quine did not consciously label himself as a naturalist in 1948, and perhaps did not even realize he had become a naturalist. For that letter omits clarity as a standard for choosing between competing theories, even though Quine had used that standard earlier in his career. And if clarity is an extra-scientific criterion, whereas 'the relative elegance and simplicity with which the theory serves to group and correlate our sense data' are scientific criteria, then the March 1948 letter proposes a properly naturalist set of theoretical virtues. If we reformulate the Naturalism Hypothesis in this way, then it is compatible with the Clarity Hypothesis.

Now the key question is: *did* Quine come to see his earlier demands for clarity as a specifically philosophical (i.e., extra-scientific) demand, and thus as not a naturalistically legitimate criterion for deciding between theories? If so, then the Clarity and Naturalism hypotheses are complementary (if not identical). Unfortunately, the answer to this question is not entirely clear. The answer seems to be: often *yes*, but sometimes *no*—though (to make matters more complex) these few 'no's are often qualified. The next sub-section discusses the equivocal status of clarity in Quine's post-nominalist writings.

4.4 Clarity's Later Fate

Thus far, I have argued that Quine in 1948 omitted clarity or naturalness from the list of nominalism's advantages, and this explains why he ceased being a nominalist then. But this leaves a significant question unanswered: after he becomes a Platonist, what does Quine think of the theoretical virtue of clarity more generally? The answer to this question is complicated and difficult. The basic interpretive problem is that clarity and its near-synonyms for Quine are often missing from Quine's later lists of theoretical virtues and theory choice, but 'clarity' or something similar nonetheless occasionally appears. However, in these comparatively rare appearances, Quine does not unequivocally endorse clarity as a theoretical virtue, or give it much weight when actually choosing between specific theories he is discussing. So it is difficult to ascertain how much of a role clarity still plays in Quine's later ontological views. The remainder of this subsection attempts to substantiate the above claims about 'clarity' in the post-nominalist Quine.

Here is a interpretive hypothesis one might propose: the reason Quine no longer considers clarity or intelligibility to be a benefit of nominalism is that he no longer considers clarity to be a theoretical virtue *simpliciter*. One piece of evidence for this is Quine's description of the relative advantages and disadvantages of nominalism versus Platonism in §48 of *Word and Object*. He writes:

> The case that emerged… for classes rested on systematic efficacy. Now it is certainly a case against nominalism's negative claims, but still it is no case against a preferential status for physical objects. In a contest for sheer systematic utility to science, the notion of a physical object still leads the field. (1960, 238)

Note that the reason Quine holds that physical objects merit 'preferential status' is *not* because they are especially clear or intuitive. Rather, they are judged according to that same standard of 'systematic efficiency,' and they are preferred because they perform so well on that score—not because of anything to do with their clarity. And later, Quine says that perhaps the nominalist "can afford to sacrifice some…of the systematic benefits of abstract objects, as offset by a twofold gain: elimination of the less welcome objects, and elimination of a drastic dualism of categories," namely the dualism between the abstract and the concrete (1960, 268). So it sounds as though the reason to adopt nominalism is its theoretical economy, both at the level of individuals and the level of categories—*not* because concreta are clearer than abstracta.

The second piece of evidence that post-nominalist Quine no longer considers clarity a virtue is that his later lists of theoretical virtues almost never include 'clarity' (or 'intelligibility' or 'intuitiveness'). Here is one list of criteria for theory choice from 1955: "simplicity... familiarity of principle... scope... fecundity" and consistency of predictions with observations (1955/1976b, 247). Later, writing with Ullian (Quine and Ullian 1970, Ch. 5), Quine's list is similar: conservatism, generality, simplicity, refutability, modesty, and empirical adequacy. The list is even shorter in Quine's 1949 lecture "Animadversions on the Notion of Meaning":

> [T]he system as a whole must conform to experience along the periphery; but discontinuities can be repaired each by any of many changes of the system. We choose by two canons: 1) maximum elegance of the whole system, 2) maximum conservationism. (2008, 155)

Here Quine only countenances three virtues: empirical adequacy, elegance, and conservativeness (later called the 'maxim of minimum mutilation').[19] So, looking at these lists of criteria for theory choice, one might conclude that the reason Quine stops thinking nominalism enjoys the theoretical virtue of clarity is that he stops thinking clarity is a theoretical virtue in general.[20]

However, at least two batches of textual evidence suggest Quine never fully abandoned clarity as a theoretical virtue. First, post-nominalist Quine sometimes still cites clarity as a theoretical virtue. For example, "[t]he same motives that impel scientists to seek ever simpler and clearer theories adequate to the subject matter of their special sciences are motives for simplification and clarification of the broader framework shared by all the sciences" (1960, 161). And Platonist Quine makes at least one list of theoreti-

[19]As multiple readers pointed out to me, one might think that, for Quine, the earlier criteria of clarity and naturalness were 'transformed' into the later virtues of conservativeness or familiarity. This may well be true, but if it is, I think that transformation should be considered a genuine change, and not merely a re-labeling: presumably, there can be newly-introduced concepts that are clear, and long-standing, familiar concepts that are unclear.

[20]Or at least, Quine ceases thinking that clarity is an *instantiated* theoretical virtue. Perhaps we could think of the March 1948 letter to Woodger as offering the following argument: 'the concept of a physical body is clear, if any concept is. But the quantum physicists have shown that the concept of a physical body is strange. Thus no concepts are clear.' Thanks to Erich Rech for discussion of this line of thought.

cal virtues that includes "naturalness" (1990, 95), which is arguably akin to clarity or intuitiveness, as we saw in earlier sections. (However, the only other virtues on the particular list containing "naturalness" are "simplicity" and "economy," so it is possible that "naturalness" is doing duty here for one of the other virtues on Quine's longer lists above.) And perhaps most explicitly, Quine's "Facts of the Matter" opens with the following invocation of clarity: "[i]t was emphasized by rationalists and empiricists alike that inquiry should begin with clear ideas. I agree about the clarity... Philosophical inquiry should begin with the clear" (1979, 155).[21]

Second, post-nominalist Quine still appeals to clarity when he evaluates particular theories. For example, in *Word and Object*, Quine writes (when praising Wiener's definition of 'ordered pair'): in a proper "analysis" or "explication", "[w]e fix on the particular functions of the unclear expression that make it worth troubling about, and then devise a substitute, clear and couched in terms to our own liking, that fills those functions" (1960, 258-259). In his autobiography, Quine describes his "critique of meaning and intensions" as "one bid for clarity", and his "insistence on the distinction between use and mention" as another (1985, 85). Elsewhere, he says that, for a concept to be clear, extensionality is a necessary but not sufficient condition (1995, 90).

Elsewhere in *Word and Object*, Quine presents an appeal to naturalness. Because the logical theories proposed to avoid Russell's paradox are artificial and ad hoc,

> [n]aturalness, for whatever it is worth, is of course lost; a multitude of... mutually incompatible systems of class theory arises, each with only the most bleakly pragmatic claims to attention. Insofar as a leaning or tolerance toward classes may have turned on considerations of naturalness, nominalism scores. (1960, 268)

So Quine here does cite 'naturalness'—the opposite of ad hoc and artificial—as an advantage of nominalism. However, Platonist Quine's invocation of naturalness on behalf of the nominalist here is complicated by the fact that he does not commit to the significance of naturalness in this quotation: note the 'for whatever it is worth' and 'insofar as tolerance toward classes turned on naturalness' qualifications here. He is not obviously endorsing naturalness as a theoretical virtue in propria persona. So

[21] After reading this opening, one might hope that Quine would offer a characterization of clarity in this piece; unfortunately, he only explicates how to measure the clarity of occasion sentences (and mathematical claims are the polar opposite of occasion sentences).

it is possible that when later Quine explicitly appeals to clarity or naturalness, at least sometimes he is doing so merely to be dialectically fair to his opponent. That is, the nominalist (specifically, Quine himself circa 1947) complains that the type restrictions needed to save non-nominalist mathematics are unnatural, so Platonist Quine feels the need to present that complaint, without himself endorsing naturalness as a genuine theoretical virtue, merely so that a full list of purported advantages of nominalism is on the table.

This is not the only time post-nominalist Quine exhibits a mixed or equivocal stance towards the significance of clarity and/or naturalness. When he introduces clarity as a possible criterion for theory choice, the clearer theory often loses. For example, as we saw earlier, when discussing Neo-Pythagoreanism in "Ontological Reduction and the World of Numbers," Quine writes that in reducing "talk of sets—and of all else—to talk of natural numbers, . . . there is an evident gain, since the natural numbers are relatively clear" (1964, 211). So numbers are clearer than sets, and this clarity counts as a 'gain', i.e. a positive benefit favoring a purely numerical ontology. But on the other hand, as discussed above, Quine rejects Pythagoreanism in that paper, since it fails the proxy-function requirement. So it is unclear how much weight Quine assigns to clarity in theory choice here (beyond being assigned less weight than the proxy-function requirement). Similarly, in 1979's "Facts of the Matter" (which overlaps somewhat with "Whither Physical Objects?"), Quine writes that "Bodies. . . are the paradigmatic objects, clearer and more perspicuous than others" (1979, 159). But later in the very same article, he says that in our current physical theories "at last bodies go by the board. . . the paradigmatic objects most clearly and perspicuously beheld" (1979, 164). So here is another example where clarity is introduced, but is outweighed by other considerations. Together, these cases suggest that even if Quine still considers clarity a theoretical virtue in his Platonist days, he does not weight it heavily when choosing between theories.

In sum, this subsection has argued that post-nominalist Quine's attitudes towards clarity as a theoretical virtue are complicated. Usually, clarity does not appear on his lists of theoretical virtues, even counting 'naturalness' as roughly synonymous with 'clarity'—but it does appear occasionally. Platonist Quine appears sometimes to introduce clarity or naturalness as a genuine theoretical virtue, but he does not weight it very much—for if he weighted it heavily, then presumably nominalism or Pythagoreanism could have 'won' against their competitors, since nominalism and Pythagoreanism have clearer ontologies than full classical mathematics. In any case, the post-nominalist Quine, unlike the earlier Quine, does not attribute the same significance to the virtue of naturalness, clarity, intelligibility, or whatever one calls it, as he does to the other theoretical virtues such as empirical adequacy, simplicity, and scope. In short, perhaps clarity is a theoretical virtue for post-nominalist Quine, and perhaps it isn't—and even if it is, it does not count for much when deciding between competing theories.

It is well-known that Quine was sympathetic to nominalism his entire professional life, and that he investigated it seriously and sympathetically in the 1930s and 40s. This chapter has shown that Quine also sympathetically investigated an even more unusual ontological position very briefly in 1940: neo-Pythagoreanism. Interestingly, Quine's motivations for investigating neo-Pythagoreanism and nominalism are the same: increased clarity of concepts and ontological simplicity. The final section offered an explanation of Quine's renunciation of nominalism: he relinquished nominalism because he ceased to think it enjoyed the theoretical virtue of clarity—or perhaps clarity itself was no longer an important theoretical virtue to be sought.

Appendix

Arithmetic and Nature [Oct. 23-26, 1940]

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Under this title, a book expounding a Neo-Pythagoreanism wherein the positive integers constitute the whole ontology. So far as logic and mathematics are concerned, I would try by following the reasoning of the Skolem-Löwenheim theorem to get an adequate system by *identifying* all classes with the positive integers; for, according to that theorem, the positive integers provide an adequate model for any system which has a model at all.

As to extra-logical matters, I would follow Carnap in adopting a coördinate language; but only insofar as place-times are concerned. Spatio-temporal regions would then become classes of quadruples of real numbers—hence classes of classes of ... classes of positive integers according to the constructions of *Mathematical Logic*; and those classes would reduce simply to individual positive integers, in turn, according to the approach previously mentioned.

If we take the reasoning of Skolem's proof[22] of the Skolem-Löwenheim theorem as a partial model, the identification of classes with positive integers would proceed as follows. We would select a *finite* set of axioms for set theory—say those of Bernays (*J. S. L.*, 1937),

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and conjoin them; then we would set up a corresponding formula in Skolem's second normal form—i.e., a formula in which all quantifiers stand at the beginning and every universal quantifier precedes every existential quantifier. This result *corresponds* to the original in the sense that it is fulfilled in exactly those universes in which the original is fulfilled. Then where the result is:

$$(x_1)\cdots(x_m)(\exists y_1)\cdots(\exists y_n)\ \phi(x_1\cdots x_m, y_1\cdots y_n),$$

we find n m-ary functions $f_1\cdots, f_n$ such that[23]

$$(x_1)\cdots(x_m)\ \phi(x_1,\cdots,x_m, f_1(x_1,\cdots,x_m),\cdots,f_n(x_1,\cdots,x_m)).$$

The axiom of choice tells us that there *are* such functions. Next we consider the function g: *on* positive integers *to* classes, which is determined by the

[22][*Quine's footnote*] *Logisch-Kombinatorische Untersuchungen*, pp. 4-9

[23]The following corresponds to (Skolem 1920/ 1967, 258).

following recursive conditions:[24]

$$\begin{aligned}
g(1) &= \Lambda \\
g(2) &= f_1(g(1), \cdots, g(1)) \\
&\vdots \\
g(n+1) &= f_n(g(1), \cdots, g(1)) \\
g(n+2) &= f_1(g(2), g(1), \cdots, g(1)) \\
&\vdots \\
g(2n+1) &= f_n(g(2), g(1), \cdots, g(1)) \\
g(2n+2) &= f_1(g(1), g(2), g(1), \cdots, g(1)) \\
&\vdots \\
g(3n+1) &= f_n(g(1), g(2), g(1), \cdots, g(1)) \\
g(3n+2) &= f_1(g(2), g(2), g(1), \cdots, g(1)) \\
&\vdots \\
g(2^m n+1) &= f_n(g(2), \cdots, g(2)) \\
g(2^m n+2) &= f_1(g(3), g(1), \cdots, g(1)) \\
&\vdots
\end{aligned}$$

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Next we try to define an arithmetical relationship '$x \in' y$' such that

$$(x)(y)(x \in' y \,\boldsymbol{.}\equiv\boldsymbol{.}\, g(x) \in g(y)).$$

The definition of course is to involve use neither of 'g' nor of '$\in$', but is to be accomplished rather within the given logic of positive integers.

We have not yet identified classes with numbers, nor does '$\in'$' yet provide quite the desired counterpart of '$\in$'. Identification along these lines would violate the principle of extensionality, since the function g often has the same class as value for different numbers as arguments. Our construction might perhaps be described rather as identifying *properties* with integers, in an intensional sense of 'property'. But we can define classes thus:

$$\mathsf{Class}\ x =_{\mathrm{df}} (y)((z)(z \in' x \,\boldsymbol{.}\equiv\boldsymbol{.}\, z \in' y) \supset\boldsymbol{.}\, x \geq y).$$

[24] The following is similar to (Skolem 1920/1967, 259-60).

Thus a class is any number which is the least of all its equivalent "properties". For convenience we may introduce special class variables thus:

$$(X)(\cdots X \cdots) =_{\mathrm{df}} (x)(\mathsf{Class}\, x \supset \cdots x \cdots).$$

Now, if we agree to drop the accent of '$\in'$', we have in effect the regular Bernays(-von Neumann) set theory along with our arithmetic.

Class abstraction can of course be defined contextually, much as in "New Foundations". The conditions for the "existence" of classes will depart from the "New Foundations", however, being rather those of the Bernays(-von Neumann) theory.

There are two points, theoretically, at which

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the program described above is capable of bogging down. First, we might not be able to find functions $f_1, \cdots, f_n$ of the described kind. We are assured of their existence only by the axiom of choice, and are not given any rule for constructing them. Second, having got these functions, we might still find difficulty in defining '$x \in' y$'. Even if we cannot define this in terms of a narrow preconceived set of arithmetical primitives, we should of course be content so long as we can define it on the basis of some additional arithmetical primitives of an arithmetically clear and intelligible kind; but of this possibility we have no guarantee in advance.

It is fairly easy, though, to set down a scheme constituting a sort of first approximation to the above. The set theory which this follows is not the Bernays(-von Neumann) theory, but one of a more constructionalistic kind. No line is drawn between elements (sets, in Bernays' phrase) and other classes; or, in other words, all classes are elements (sets). All of Bernays' axioms for the existence of sets carry over, except that there is no provision for the existence of complementary classes ($\bar{x}$); instead, the universal class is assumed direct.

I shall simply set forth the resulting set of definitions. As primitive or antecedently defined we are to assume, besides the truth functions

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and quantification with respect to the positive integers, the following notations:

$$1, 2, \cdots,\ x = y,\ x < y,\ x \leq y,\ x + y,\ x \cdot y,\ x^y,\ \mathsf{Prime}\, x,\ x \frown y,$$

this last being so construed that

$$(p_1^{x_1} \cdot p_2^{x_2} \cdot \cdots \cdot p_m^{x_m}) \frown (p_1^{y_1} \cdot p_2^{y_2} \cdot \cdots \cdot p_n^{y_n}) = p_1^{x_1} \cdot p_2^{x_2} \cdot \cdots \cdot p_m^{x_m} \cdot p_{m+1}^{y_1} \cdot p_{m+2}^{y_2} \cdot \cdots \cdot p_{m+n}^{y_n}$$

where p_i is the ith prime number (other than 1).

The further definitions are as follows.

$$
\begin{aligned}
x \,\mathsf{In}_z\, y &=_{\mathrm{df}} \mathsf{Prime}\ z \,.\, (\exists w)(z^x \cdot w = y \,.\, \neg(\exists v)(z \cdot v = w)).\\
x \,\mathsf{In}\, y &=_{\mathrm{df}} (\exists z)(x \,\mathsf{In}_z\, y).\\
x \,\mathsf{Pr}_z\, y &=_{\mathrm{df}} (\exists u)(\exists v)(u < v \,.\, x \,\mathsf{In}_u\, z \,.\, y \,\mathsf{In}_v\, z).\\
x; y &=_{\mathrm{df}} 2^x \cdot 3^y.\\
x \in y &=_{\mathrm{df}} (\exists z)(x; y \,\mathsf{In}\, z \,.\, (u)(v)(u; v \,\mathsf{In}\, z. \supset :\\
&\quad v = 2 \,.\vee.\, v = 2^{u+1} \,.\vee.\, v = 5 \,.\, u \,\mathsf{Pr}_z\, u; v \,.\vee\\
&\quad (\exists w)(\exists r)(\exists s)(\exists t)(v = 3 \,.\, u = 2^{w+1} \,.\vee.\, v = 5^{w+1} \,.\, (u; t); w \,\mathsf{Pr}_z\, u; v.\\
&\quad \vee.\, v = 3^{w+1} \,.\, u = s; t \;.\; s; w \,\mathsf{Pr}_z\, u; v \,.\\
&\quad \vee.\, v = 7^{w} \,.\, u = s; t \;.\; (t; s); w \,\mathsf{Pr}_z\, u; v.\\
&\quad \vee.\, v = 11^{w} \,.\, u = (r; s); t \,.\, (r; (s; t)); w \,\mathsf{Pr}_z\, u; v.\\
&\quad \vee : v = w; t : u; w \,\mathsf{Pr}_z\, u; v \,.\vee.\, u; t \,\mathsf{Pr}_z\, u; v : t < w.\\
&\quad \vee .u; w \,\mathsf{Pr}_z\, u; v \,.\, u; t \,\mathsf{Pr}_z\, u; v))).
\end{aligned}
$$

This last definition introduces what corresponds to the '$\in'$' of the previous general discussion. Then, in conformity with that general discussion, we conclude with the definition of class and of quantification with respect to classes:

$$
\begin{aligned}
&\mathsf{Class}\ x =_{\mathrm{df}} (y)((z)(z \in x. \equiv .z \in y) \supset .z \leq y).\\
&(X)(\cdots X \cdots) =_{\mathrm{df}} (x)(\mathsf{Class}\ x \supset \cdots x \cdots).
\end{aligned}
$$

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Note that we get this as a theorem:

$$(X)(Y)(X = Y. \equiv (z)(z \in X \,.\equiv.\, z \in Y)).$$

The contextual definitions of abstraction are as follows:

$$y = \hat{x}(\cdots x \cdots) =_{\mathrm{df}} \mathsf{Class}\ y \,.\, (x)(x \in y. \equiv \cdots x \cdots),$$

and then, for other immediate (quasi-atomic; cf. *Mathematical Logic*) contexts of abstracts, this scheme of definition:

$$\text{—}\hat{x}(\cdots x \cdots)\text{—} =_{\mathrm{df}} (\exists y)(y = \hat{x}(\cdots x \cdots) \,.\, \text{—}y\text{—}$$

Arbitrary rule to provide uniqueness of definitional expression: in any quasi-atomic context, attend to the first occurrence of an abstract first.

The definitions are so fashioned that the following theorems of existence of classes are presumably derivable in routine fashion.

$$\begin{aligned}
&(\exists w)(w = \hat{x}\neg(x = x)),\\
&(\exists w)(w = \hat{x}(x = x)),\\
&(\exists w)(w = \hat{x}(\exists y)(x = \hat{z}(z = y))),\\
&(\exists w)(w = \hat{x}(\exists y)(\exists z)(x = y;z \,\centerdot\, y \in z)),\\
&(y)(\exists w)(w = \hat{x}(x = y)),\\
&(y)(\exists w)(w = \hat{x}(\exists z)(x;z \in y)),\\
&(y)(\exists w)(w = \hat{x}(\exists u)(\exists v)(x = u;v \,\centerdot\, u \in y)),\\
&(y)(\exists w)(w = \hat{x}(\exists u)(\exists v)(x = u;v \,\centerdot\, v;u \in y)),\\
&(y)(\exists w)(w = \hat{x}(\exists z)(\exists u)(\exists v)(x = (z;u);v \,\centerdot\, z;(u;v) \in y)),\\
&(y)(z)(\exists w)(w = \hat{x}(x \in y \,\centerdot\!\vee\!\centerdot\, x \in z)),\\
&(y)(z)(\exists w)(w = \hat{x}(x \in y \,\centerdot\, x \in z)).
\end{aligned}$$

Abbreviated through adoption of definitions such as appear in *Mathematical Logic*, these theorems take the following forms:

$$(\exists w)(w = \Lambda),$$

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$$\begin{aligned}
&(\exists w)(w = \mathrm{V}),\\
&(\exists w)(w = \text{Unity}),\ \text{[not to be confused with integer 1]}\\
&(\exists w)(w = \mathcal{E}^{25}),\\
&(y)(\exists w)(w = \iota y),\\
&(y)(\exists w)(w = \hat{x}(\exists z)y(x,z)),\\
&(y)(\exists w)(w = \hat{u}\hat{v}(u \in y),\\
&(y)(\exists w)(w = \breve{y}),\\
&(y)(\exists w)(w = \hat{x}\hat{v}(\exists z)(\exists u)(x = z;u \,\centerdot\, y(z,u;v))),\\
&(y)(z)(\exists w)(w = y \cup z),\\
&(y)(z)(\exists w)(w = y \cap z).
\end{aligned}$$

Perhaps, by following more or less the lines followed by Bernays (*J. S. L.*, 1937), it can be shown that the closure of every matrix of the form

$$(\exists w)(w = \hat{x} \cdots)$$

[25] This symbol denotes the membership relation.

is a theorem where the blank contains no occurrences of 'w' and no truth-functions except alternation and conjunction (on expansion into primitive notation).

Does this exclusion of denial from abstracts result in an intolerably restricted set theory? If not, this theory has—besides the virtues suggested on p.224—the virtue of resolving the logical paradoxes without artificial or ad hoc postulation; for the postulates of the present logic will all be intuitively acceptable postulates about intuitively clear notions, viz. positive integers. The theory of membership, being merely an abbreviation of part of this theory of integers, is *explained* rather than posited ad hoc; and whatever there is of strangeness in its laws is traceable direct to the definitions, rather

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than merely to arbitrary assumptions. These same remarks would of course apply to any other theory constructed along the general lines of the present article.

Insofar as irrational numbers admit of definition within a theory of this sort, the theory may be viewed as a Pythagorean predicament of incommensurability.

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